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Kalibrering af partielle sikkerhedsfaktorer for udmattelse af vindmøllerotorer

Bilagsrapport

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Denne bilagsrapport indeholder det tekniske baggrundsmateriale til rapporten *Kalibrering af partielle sikkerhedsfaktorer for udmattelse af vindmøllerotorer,* Risø-R-1204(DA). Bilagsrapporten består af fem enkeltrapporter, der tidligere er publiceret i DNV-regi. Rapporterne er her samlet for komplettere analyserne og give læseren et fyldestgørende billede af de benyttede modeller.

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REPORT

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Summary

A probabilistic model for evaluation of the safety of a wind-turbine rotor blade against fatigue failure in flapwise bending is presented. The model accounts for uncertainties in load and resistance. The model is applied in conjunction with a first-order reliability method to perform a structural reliability analysis of a particular, site-specific wind turbine. The turbine selected for this purpose is a 500 kW wind turbine. The probability of fatigue failure in flapwise bending of one of the rotor blades of this wind turbine over a twenty-year design life is calculated. It is demonstrated how the reliability analysis results can be used to calibrate partial safety factors for load and resistance for use in conventional deterministic fatigue design.

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97-2048	B3, B4, F1, K0

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CALIBRATION OF PARTIAL SAFETY FACTORS FOR DESIGN OF WIND-TURBINE ROTOR BLADES AGAINST FATIGUE FAILURE IN FLAPWISE BENDING – 500 KW TURBINE

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1. INTRODUCTION

Wind-turbine rotor blades exposed to wind loading are vulnerable to cumulative fatigue damage owing to the cyclic nature of the loading. The wind speed that causes bending of the rotor blades exhibits a natural variability, such that the load amplitudes become random, and the *S–N* curve that gives the number of stress cycles to failure and represents the resistance of the rotor blade material is encumbered with uncertainty owing to a limited number of test specimens as well as variability from one specimen to another.

Partial safety factors are used in structural design as factors on characteristic values of governing load and resistance quantities to account for variabilities and uncertainties in these quantities.

This report demonstrates how a structural reliability method can be applied as a rational means to analyze a wind-turbine rotor blade with respect to fatigue in flapwise bending, and to establish partial safety factors for design of such rotor blades against fatigue failure. A site-specific wind turbine of a prescribed make is considered, here a 500 kW turbine with 19 m long rotor blades, and probabilistic models for the wind loading and its transfer to bending stresses in the rotor blades is established together with a stochastic representation of the material resistance. The event of fatigue failure in flapwise bending is considered as based on a Miner's sum formulation for cumulative damage.

2. THEORY FOR LOAD, RESISTANCE, AND CUMULATIVE DAMAGE

2.1 Wind Climate and Load History for Rotor Blade

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the turbulence intensity I_T. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \tag{1}$$

in which k and A are site- and height-dependent coefficients. The turbulence intensity I_T is also site- and height-dependent. It is defined as the standard deviation of the wind speed divided by the mean wind speed U_{10} and represents the gustiness of the wind about this mean. The turbulence intensity I_T is here assumed to be independent of U_{10}, but could in general be modelled as dependent on U_{10} which would be the case for a site with inhomogeneous terrain. Detailed information about the distribution of I_T is not available. The mean value can be taken as

$$E[I_T] = \left(\ln\frac{z}{z_0}\right)^{-1} \tag{2}$$

where z is the height above the ground, i.e., the hub height of the rotor, and z_0 is the roughness parameter for the terrain. A representative value of the coefficient of variation is COV=0.25, and the distribution type can be assumed to be lognormal. The (U_{10},I_T) space is discretized into a number of bins ("two-dimensional intervals") of approximately constant values of U_{10} and I_T.

One rotor blade is considered in the following. Let X denote the bending moment range at the blade root in flapwise bending. Hence, X is the double amplitude of the flapwise bending moment response owing to an aerodynamic load cycle that is applied to the rotor blade. One bending moment range is associated with each load cycle, and load cycles are identified by rainflow counting. Observations of the bending moment range X are recorded in 10-minute intervals. Each 10-minute record of X is binned by U_{10} and I_T. For a particular bin (U_{10},I_T) there will be M 10-minute records of X, and they are used to give an estimate of the long-term distribution of X conditioned on (U_{10},I_T), i.e., $X|(U_{10},I_T)$, on discretized form. The number of load cycles n_{10} in each 10-minute interval is also observed and depends on U_{10} and I_T.

Because the distribution of $X|(U_{10},I_T)$ is encumbered with uncertainty owing to limited data for its estimation, it is desirable to parametrize the distribution and represent this uncertainty in reliability analyses as uncertainty in the distribution parameters. The distribution of $X|(U_{10},I_T)$ can be parametrized in terms of its statistical moments. The first three statistical moments are used for this purpose. These moments are the mean value a_1, the standard deviation a_2, and the skewness a_3. Their expected values $E[a_i]$, i=1,2,3, can be estimated based on the observed discretized version of the conditional distribution of $X|(U_{10},I_T)$. Their standard deviations $D[a_i]$, i=1,2,3, and also their correlation matrix ρ can be estimated by a resampling technique such as the jackknife or the bootstrap, see Efron and Tibshirani (1993).

It is assumed that the expected values and standard deviations of a_1, a_2, and a_3 conditioned on U_{10} and I_T are adequately represented by the polynomial surfaces

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^2 + b_{3i}I_T + b_{4i}I_T^2 \tag{3}$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^2 + c_{3i}I_T + c_{4i}I_T^2 \tag{4}$$

in which the coefficients b_{ji} and c_{ji}, j=0,...4, i=1,2,3, are determined by least-squares regressions of all estimated mean values and standard deviations of a_1, a_2, and a_3 over the (U_{10},I_T) space.

Based on the central limit theorem and the assumption that the correlation matrix ρ is independent of (U_{10},I_T), the three moments a_1, a_2, and a_3 can be represented as

$$a_i = E[a_i] + U_i D[a_i], \quad i = 1,2,3 \tag{5}$$

in which $\mathbf{U}=(U_1, U_2, U_3)^T$ is a three-dimensional normally distributed variable with zero mean, unit variance, and correlation matrix ρ. Note in this context that U_i is standard notation for a standard normally distributed variable within the field of structural reliability and is not to be confused with any wind speed. Note also that the vector $\mathbf{U}=(U_1, U_2, U_3)^T$ represents the statistical uncertainty in the three moments a_1, a_2, and a_3 owing to the limited data available for their estimation.

Above, the statistical moments a_1, a_2, and a_3 of the available measured data for the bending moment range X at the blade root in flapwise bending have been dealt with. However, no statement has so far been made with respect to the distribution of the bending moment amplitudes themselves, neither in the short term, conditional on a particular wind climate (U_{10},I_T), nor in the long term such as over the design life of the rotor blade. A model for the distribution of the bending moment range X is therefore dealt with in the following.

Load response amplitudes are often seen to have marginal distributions which are close to Weibull distributions. The bending moment range is two times such a load response amplitude. Based on the first three moments a_1, a_2, and a_3 of the distribution of the conditional bending moment range $X|(U_{10},I_T)$, this distribution can be modelled as a quadratic expansion of a parent Weibull-distributed variable U_W. The parent Weibull-distributed variable U_W is chosen such that it has the same mean a_1 and the same standard deviation a_2 as the distribution of $X|(U_{10},I_T)$ which is to be modelled. For the case that the skewness of U_W is smaller than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a softening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa(U_W + \varepsilon U_W{}^2) \tag{6}$$

For the case that the skewness of U_W is greater than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a hardening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa\frac{\sqrt{1+4\varepsilon U_W} - 1}{2\varepsilon} \tag{7}$$

In both cases, the model is referred to as a quadratic Weibull model, and the coefficients ε, κ, and x_{min} are chosen such that the mean value a_1, the standard deviation a_2, and the skewness a_3 of the distribution of $X|(U_{10},I_T)$ are all preserved. The quadratic Weibull model may be thought of as a generalized or distorted Weibull distribution. Reference is made to Lange and Winterstein et al. (1996). Note that the quadratic Weibull distribution provides a better fit to the data, and in particular a better representation of the important upper tail of the distribution, than the distorted lognormal distribution used by Ronold et al. (1994) in a first approach to a parametrized representation of the load range distribution. The distorted lognormal distribution, obtained by a logarithmic Hermite polynomial expansion of a parent Gaussian distribution, is known to have a heavy upper tail which, as commented by Ronold et al. (1994), leads to overprediction of upper quantiles of the bending moment ranges and thereby of the high-range stresses. Note also that the quadratic Weibull model provides results very close to those obtained by a cubic Weibull model which preserves the first four statistical moments of the distribution of $X|(U_{10},I_T)$, including the kurtosis a_4, but which is computationally much more cumbersome and time-consuming and therefore less attractive, see Ronold et al. (1996). The accuracy of predictions made by means of the quadratic Weibull models of Eqs. (6) and (7) is further dealt with later.

The section modulus at the rotor blade at the blade root is W, and the stress range S corresponding to the moment range X is $S=X/W$. When a discretization of the stress range space is introduced, either Eq. (6) or Eq. (7), depending on the value of the skewness a_3, can be applied in conjunction with the distribution function of the parent Weibull variable U_W to calculate the probability content of each interval ΔS of this discretization. The corresponding number of cycles within each such interval in a 10-minute period can be determined as this probability content times the total number of cycles $n_{10}(U_{10},I_T)$.

Integrating contributions from all possible 10-minute wind climate bins (U_{10},I_T), weighted according to the quoted Weibull distribution for U_{10} and the lognormal distribution for I_T, this can be used to establish an ordered history of the stress range S over the design life T_L of the rotor blade. This compound lifetime distribution of the stress range S can be expressed in terms of the number of stress cycles n whose associated stress range exceeds a level S during the design life of the rotor blade, see Figure 1. This, in turn, can be used to calculate the number of stress cycles Δn within an interval ΔS of the discretization of the stress range space.

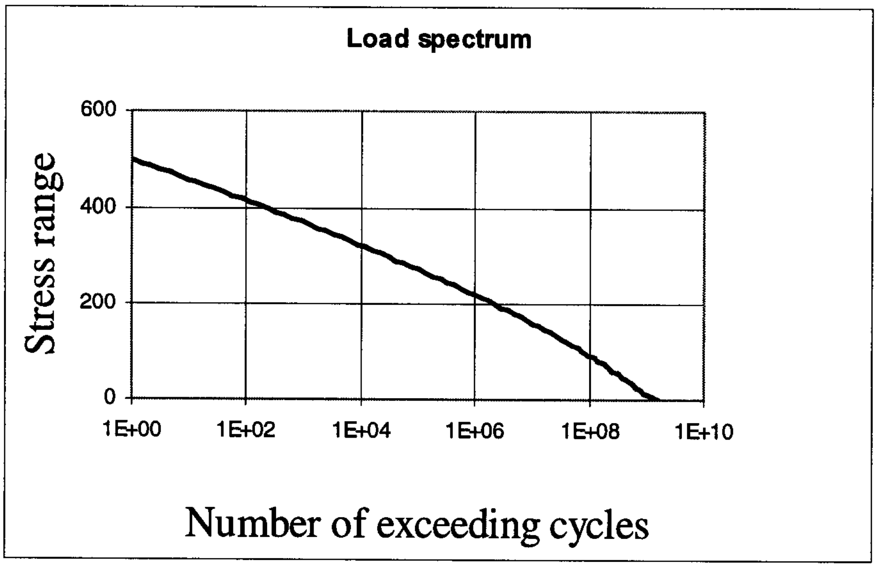

Figure 1 Example of compound stress range distribution over a design life

The number of cycles $n_{10}(U_{10},I_T)$ in a 10-minute interval is also encumbered with uncertainty. However, the coefficient of variation is inversely proportional with the square-root of the interval length, such that when $n_{10}(U_{10},I_T)$ is scaled to give the number of cycles over a long time span such as a 20-year design life, the uncertainty in this number becomes insignificant and can be ignored. In the reliability analysis $n_{10}(U_{10},I_T)$ is therefore left as a deterministic quantity only dependent on U_{10} and I_T.

2.2 Fatigue Strength and *S–N* curve

For a given stress range S, the number of cycles N to failure is generally expressed through an S–N curve, $N=BS^{-k}$. However, in tests of composite materials for use in rotor blades, the strain amplitude ε is usually measured rather than the stress range S. Hence, for such materials the number of cycles N to failure is expressed through an ε–N curve. This curve can be expressed by the following relation

$$\log_{10} N = \log_{10} K - m\log_{10} \varepsilon \tag{8}$$

in which K and m are coefficients. This gives a linear model for $\log_{10}N$

$$\log_{10} N_i = \log_{10} K - m\log_{10} \varepsilon_i + e_i \ , i=1,...n \tag{9}$$

in which the pair ($\log_{10}K$, m) describes the expected behavior and can be estimated by a linear regression analysis based on n observed data pairs (ε_i,N_i). The zero-mean terms e_i denote residuals that represent local variations from test specimen to test specimen, or from one point of the rotor blade to another. The standard deviation σ_e of the residuals e_i will result as a byproduct of the regression analysis, and so will the standard deviations and correlation coefficient of $\log_{10}K$ and m. The stress range S that corresponds to the strain amplitude ε can be expressed as S=2$E\varepsilon$, where E denotes the modulus of elasticity of the material in the direction of the loading. The modulus of elasticity is idealized as a constant here, but may in general vary with the magnitude of the strain. A refined representation with such a variation included would be desirable. Further, the possible effect of a non-zero mean stress has been ignored, mainly because of limitations in available test data. Scale effects from test specimen to prototype and long-term environmental degradation effects owing to exposure to moisture and ultraviolet light have been left out of consideration. Also the inclusion of such effects would be desirable if data would permit.

2.3 Cumulative Damage and Failure Criterion in Fatigue

According to Miner's rule, fatigue failure in a structural material is defined to occur when the accumulated damage D exceeds 1.0, where D is defined as

$$D = \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{10}$$

Here, Δn is the number of load cycles at stress range S in the lifetime of the rotor blade, and N is the number of cycles to failure at this stress range. The sum is over all stress ranges S_i in a sufficiently fine discretization of the stress range space.

3. PROBABILISTIC AND DETERMINISTIC MODELLING

The reliability of a site-specific wind turbine against fatigue failure of one of its rotor blades in flapwise bending is considered. The reliability is computed by a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The input to the reliability analysis consists of a limit state function, specified in terms of a set of basic variables which consist of stochastic variables as well as deterministic parameters. Furthermore, the statistical distributions of the stochastic variables must be given, and the values of the deterministic parameters must be specified. The following sections describe the stochastic variables, the deterministic parameters, and the limit state function. Separate sections are devoted to make and site of wind turbine, environmental loading, fatigue strength, model uncertainty, and limit state function.

3.1 Wind Turbine Characteristics

A 500 kW wind turbine is considered. The hub height is z=35 m, and the section modulus of the rotor blade at the blade root in flapwise bending is taken as W=0.0015 m^3.

3.2 Environmental Loading

The wind turbine is considered for a location whose wind loading regime is characterized by a scale parameter A=9.1 m/sec, a slope parameter k=1.9, and a terrain roughness z_0=0.05 m, the latter thus implying a mean value of the turbulence intensity of $E[I_T]$ =0.153.

A total of 1183 10-minute records of flapwise bending moment ranges X for various wind climate realizations (U_{10},I_T), considered fixed within each 10-minute interval, are available. For each bin (U_{10},I_T), the available M 10-minute moment range records are merged, and the observed moment ranges X are sorted in increasing order. From this, the cumulative distribution function of $X|(U_{10},I_T)$ is derived, and its first three moments, here denoted a_1 through a_3, are estimated. The standard deviations of these three moments are obtained by jackknifing the M 10-minute records. As stated in a previous section, the following model is chosen to represent the coefficients a_1 through a_3

$$a_i = E[a_i] + U_i D[a_i], \qquad i = 1,2,3 \tag{11}$$

in which the mean value $E[a_i]$ and standard deviation $D[a_i]$ of the ith moment a_i are represented as

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^2 + b_{3i}I_T + b_{4i}I_T^2 \tag{12}$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^2 + c_{3i}I_T + c_{4i}I_T^2 \tag{13}$$

Table 1 Estimated Coefficients in Polynomial Model for $E[a_i]$

i	b_0	b_1	b_2	b_3	b_4
1	-5.0568	1.5547	-0.0092	244.10	-575.08
2	4.6875	0.7266	0.0101	228.50	-551.22
3	1.6387	0.0249	-0.0035	1.6770	-5.7838

Table 2 Estimated Coefficients in Polynomial Model for $D[a_i]$

i	c_0	c_1	c_2	c_3	c_4
1	2.2512	-0.0863	0.0069	-20.740	72.463
2	2.0024	0.0591	-0.0026	-20.752	64.379
3	0.0822	0.0107	-0.0006	-1.1925	3.4608

The polynomial coefficients in these expressions are determined by a least-squares regression of the available data and are presented in Tables 1 and 2. The stochastic variables denoted $\mathbf{U}=(U_1,U_2,U_3)^T$ represent the statistical uncertainties in the bending moment range distributions and follow a three-dimensional normal distribution with mean values 0.0, standard deviations 1.0, and a correlation matrix which is estimated to be

$$\rho=\begin{bmatrix} 1.000 & 0.937 & -0.346 \\ 0.937 & 1.000 & -0.210 \\ -0.346 & -0.210 & 1.000 \end{bmatrix} \tag{14}$$

The number of aerodynamic stress cycles n_{10} in a 10-minute interval is represented as a function of (U_{10},I_T) as follows

$$n_{10}=1704-102.8U_{10}+7.027U_{10}^2-10.08I_T+1402I_T^2 \tag{15}$$

in which the coefficients are estimated by a least-squares regression from a total of 1183 records of n_{10}, when U_{10} is quoted in units of m/sec.

As described in a previous section, the conditional distribution of $X|(U_{10},I_T)$, expressed in terms of a parent Weibull distribution, is used in conjunction with the long-term distributions of U_{10} and I_T as well as the number of cycles $n_{10}(U_{10},I_T)$ in 10-minute intervals to establish an ordered lifetime history of the bending moment range X for the considered rotor blade in flapwise bending. The corresponding history of the bending stress range S of interest for prediction of cumulative damage by Miner's sum is easily derived by division by the section modulus W, hence $S=X/W$.

3.3 Resistance and Stiffness of Composite Laminate

As stated in a previous section, the ε–N curve that gives the number of stress cycles N to failure as a function of the strain amplitude ε is given by the linear relationship

$$\log_{10} N = \log_{10} K - m\log_{10}\varepsilon + e \tag{16}$$

for the rotor blade laminate.

A total of 81 observed pairs (ε,N) are available from laboratory tests on specimens of a polyester laminate reinforced by five layers of combined woven glass roving and chopped strand mat with fibres oriented at 0/90 during testing and with some fibres in the load direction. A regression analysis of these data pairs according to the linear model leads to the following estimates of mean values, standard deviations, and correlation coefficient for the coefficients $\log_{10} K$ and m

$$E\begin{bmatrix}\log_{10} K \\ m\end{bmatrix}=\begin{bmatrix}-12.372 \\ 7.912\end{bmatrix} \qquad D\begin{bmatrix}\log_{10} K \\ m\end{bmatrix}=\begin{bmatrix}0.513 \\ 0.247\end{bmatrix} \qquad \rho=-0.996 \tag{17}$$

when strain amplitudes are quoted as a dimensionless absolute quantity. Under the central limit theorem the distribution of $(\log_{10}K,m)$ is a bivariate normal distribution. The standard deviation of the residual term e is estimated to be σ_e=0.396. The zero-mean residual term e is represented by a normal distribution with this standard deviation. For details about the data and the tests of the laminates, reference is made to Echtermeyer et al. (1993) and Echtermeyer (1994).

A constant modulus of elasticity is used, E=29.7·10^6 kPa.

3.4 Model Uncertainty

Model uncertainty can be associated with all simplifications and idealizations made in the formulation of the engineering models that are used for analysis of fatigue damage and failure of a rotor blade in bending. One of these model uncertainties is considered here, namely that which is associated with the use of the quadratic Weibull model for representation of the distribution of the bending moment range conditional on the wind climate (U_{10},I_T). Damage predictions by the Miner sum, based on such quadratic Weibull distributions for the loading, are therefore multiplied by a random factor F_M. This random factor represents the bias and uncertainty in these damage predictions as associated with the use of the quadratic Weibull model for the conditional load distributions. For a series of 39 wind climate bins (U_{10},I_T), observations of conditional load distributions are available, and the corresponding quadratic Weibull models for these distributions have been fitted. For each bin, two damage predictions by the Miner sum have been made, the first based on the observed empirical load distribution, the second based on the fitted quadratic Weibull model, and the ratio between the two predictions has been calculated. A statistical analysis of the 39 damage ratios gives the following estimates of the mean value and standard deviation of the random model uncertainty factor F_M

$$E[F_M]=0.575 \qquad D[F_M]=0.171 \tag{18}$$

The distribution of F_M is taken as a normal distribution.

It appears that a significant bias in the damage predictions by the quadratic Weibull model for the loading is present and accounted for by a mean value of F_M which is considerably less than 1.0. However, because the fatigue damage is very nonlinear with respect to the loading, this bias corresponds only to a mere 5-7% overprediction, on average, of the bending moment ranges by the quadratic Weibull model. This may serve to support use of the quadratic Weibull model as a fairly accurate model for representation of flapwise bending moment ranges of rotor blades, in particular when considering it is based on a fit to the first three statistical moments only.

3.5 Limit State Function

The reliability against fatigue failure of the considered rotor blade in flapwise bending is analyzed for the cyclic loading caused by wind over the design life. For this purpose, a limit state function is defined

$$g(\mathbf{X}) = 1 - F_M D(\mathbf{X}) = 1 - F_M \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{19}$$

in which D is the predicted cumulative fatigue damage expressed through the Miner's sum as defined in a previous section, and $\mathbf{X}$ denotes the vector of stochastic variables which include the variables **U** that represent the uncertainty in the loading, the variables ($\log_{10}K,m,e$) that represent the uncertainty in the resistance, and the variable F_M that represents the bias and uncertainty in the cumulative damage predictions as resulting from use of the quadratic Weibull model for the loading.

4. RELIABILITY ANALYSES

The reliability is the complement of the failure probability

$$P_F = P[g(\mathbf{X}) \le 0] \tag{20}$$

and may be expressed in terms of the reliability index $\beta=-\Phi^{-1}(P_F)$. The reliability is computed by means of a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The probabilistic analysis program PROBAN, see Tvedt (1989), is used for this purpose. The results of the reliability analysis are presented in Table 3.

Table 3 Results of Reliability Analysis
20-Year Lifetime Fatigue in Flapwise Bending
Rotor Blade, W=0.0015 m^3

Probability of Failure $P_F = 0.63 \cdot 10^{-4}$
Reliability Index β=3.84

Variable	Distribution	Design point $\mathbf{x}^*$	Importance factor α^2
U_1	Normal	0.4902	
U_2	Normal	0.5240	0.020
U_3	Normal	0.0031	
$\log_{10}K$	Normal	-11.776	0.111
m	Normal	7.6151	
e	Normal	-1.3732	0.815
F_M	Normal	0.7274	0.054

By examination of the resulting importance factors reported in the fourth column of Table 3, it appears that the inherent variability in the fatigue life as represented by the uncertainty in the residual e of the ε–N curve is by far the single most important uncertainty source. As much as 82% of the total uncertainty importance is attributed to this resistance variable, while the other ε–N curve variables m and $\log_{10}K$ vouch for another 11% of the uncertainty importance. This leaves as little as 2% uncertainty importance ascribed to the uncertainty in the load variables U_1, U_2, and U_3, and 5% ascribed to the load model uncertainty factor F_M.

This represents a significant shift in the uncertainty importance from the fifty-fifty split between load and resistance variables reported by Ronold et al. (1994) for a similar wind turbine. This shift in the relative importance between the various uncertainty sources is attributed to the fact that the present study capitalizes on a distorted Weibull distribution for the bending moment ranges, which is believed to provide a good representation of reality, while the study by Ronold et al. (1994) is based on a distorted lognormal distribution for these ranges. This distorted lognormal distribution model is in conformance with what was state-of-the-art at the time of that study, but is prone to overestimate the probability content in the upper tail of the distribution of the bending moment ranges with overprediction of the higher bending stresses (and thereby of the uncertainty importance factor for loads) as the result. The replacement of the distorted lognormal load distribution of the study by Ronold et al. (1994) by the distorted Weibull distribution of the present study is considered to represent a significant improvement and is well in conformance with other recent studies, see Lange and Winterstein (1996). On this background, the observed reduction in the uncertainty importance factor for the loads by this replacement is a natural finding, and it emphasizes the importance of a proper representation of the load distribution and in particular its upper tail.

An inspection of the computational results reveals that the major contribution to the cumulative damage is ascribed to the about 10^7 medium-amplitude stress cycles in a couple or more cycle-number decades centered about $\log_{10}N=6$ in the bending moment range distribution. This is a fairly small fraction of the total of about 10^9 stress cycles that occur over the design life of the rotor blade. The fact that such a low fraction of the applied stress cycles vouches for most of the cumulative damage can be ascribed to the value of the slope parameter m of the ε–N curve which is approximately equal to 8 for the composite laminate in the present case. Stress ranges are raised to the mth power for prediction of the number of cycles to failure. This implies that the higher the value of m, the more dominant are the high stress ranges. A comparison can be made with welded steel details whose m values are usually in the range 3-4 and whose major contribution to accumulated fatigue damage is ascribed to the low-ampli-tude stress cycles that correspond to the lower right part of the stress range distribution in Figure 1. These stress cycles form the majority of the total number of stress cycles over the design life. An interesting consequence of this dependency of the cumulative damage on the value of m is that if epoxy materials are considered for the rotor blade, for which m values of up to 12 or 13 are seen, the fatigue problem may be turned into an extreme value problem as far as the loading goes. Even for the present m value of 8, this indicates how important a proper estimation of the upper tail of the load distribution is.

5. CALIBRATION OF PARTIAL SAFETY FACTORS

5.1 Philosophy

It is of interest to demonstrate how reliability analysis results, obtained as outlined in the previous chapters, play a role in codified practice and design. Calibration of partial safety factors for design is an important application. With the first-order reliability method available, it is possible to determine sets of equivalent partial safety factors which result in rotor blade designs with a prescribed reliability. As a first step, a target reliability index β_t must be selected.

The choice for the target reliability index can be derived from a utility-based feasibility assessment in a decision analysis, or by requiring that the safety level as resulting from the design by a reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society. A range of target reliability indices will be considered in the following.

In the case of a prescribed reliability level which is different from the one that results from an actually executed reliability analysis of a wind turbine rotor blade, the geometrical quantities of the blade have to be adjusted such that this required reliability level results from a reliability analysis of the modified blade. The geometrical quantities which can be adjusted to achieve a specified reliability level are sometimes denoted design parameters. It is most practicable to operate on just one such design parameter when adjusting the design in order to reach the specified reliability level. For a rotor blade, the most practicable parameter to adjust is the section modulus W which is a function of the cross-sectional properties of the blade at the root.

To cover a range of target reliabilities, a series of reliability analyses is carried out for a range of values for the section modulus W. This gives the reliability index β as a function of the section modulus W, $\beta=\beta(W)$. In addition, as a byproduct of the reliability analyses, the values of the design point $e_d{}^*$ of the ε–N curve residual e prove useful in the following and are retained.

5.2 Characteristic Values of Governing Variables

Characteristic values have to be selected for the governing load and resistance variables. For design in ultimate loading, the 98% quantile of the annual maximum load is traditionally used as the characteristic load value. For design in fatigue loading, this has hardly any meaning, as a characteristic load distribution for the design life needs to be selected rather than such a single characteristic load value. The governing load distribution is a compound distribution of the bending moment range over the design life of the rotor blade and has contributions from many moment range distributions conditioned on different 10-minute wind climates (U_{10},I_T). For simplicity, an idealized characteristic moment range distribution is adopted here,

$$X = k_R X_0 \left(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}\right) \tag{21}$$

in which X denotes the moment range which is exceeded in n stress cycles during the design life T_L, $N_r=f_rT_L$ is the number of rotor cycles in this life, f_r denotes the rotor frequency, X_0 is a characteristic bending moment whose derivation is described below, and k_R is a scaling factor. A value of k_R =2.2 is applied in the following. This value has been calibrated as the one that will lead to a cumulative damage approximately equal to the median damage for the true compound moment range distribution derived from the observed conditional moment range distributions as described above.

In accordance with the Danish code, see Dansk Ingeniørforening (1992), the characteristic bending moment is defined as

$$X_0 = \frac{\rho}{2} w^2 c C_L \frac{R^2}{3} \tag{22}$$

in which R is the length or radius of the rotor blade measured from the center of the rotor to the tip, c is characteristic chord length at $2R/3$, C_L=1.5 is a lift coefficient at $2R/3$, ρ=1.28 kg/m^3 is the density of air, and w is a reference wind speed defined by

$$w^2 = (\frac{4\pi}{3} f_r R)^2 + v_0{}^2 \tag{23}$$

where f_r is the rotor frequency as before, and v_0 is the 10-minute mean wind speed at stalling of the entire rotor blade. For the considered 500 kW wind turbine, the following numbers apply: T_L=20 years, f_r =0.5033 sec^{-1}, N_r =0.317·10^9 rotations, R=19.5 m, c=1.18 m, and v_0=14.6 m/sec.

For ε–N curves, it is a standard approach to select the characteristic ε–N curve as the curve that results when the estimated ε–N curve is shifted to the left by a distance equal to two times the standard deviation of the residual e, hence

$$\begin{aligned}\log_{10} N &= E[\log_{10} K] - E[m]\log_{10}\varepsilon - 2\sigma_e \\ &= -12.372 - 7.912\log_{10}\varepsilon - 2\cdot 0.396 \\ &= -13.164 - 7.912\log_{10}\varepsilon\end{aligned} \tag{24}$$

when values for the considered rotor blade laminate are substituted. Reference is made to Det Norske Veritas (1984) and Dansk Ingeniørforening (1992).

5.3 Partial Safety Factors and Design Load and Resistance Properties

Two partial safety factors are introduced. A load factor γ_f greater than 1.0 is applied as a factor on all load values of the characteristic moment range distribution and the relation $S=X/W$ between stress range S and moment range X is substituted such that the design stress range distribution becomes

$$S = \gamma_f k_R \frac{X_0}{W}(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}) \tag{25}$$

Correspondingly, γ_m is a material factor greater than 1.0. For any given number of cycles to failure, the characteristic strength is divided by this number to give the design strength. Recall the relationship $S=2E\varepsilon$ between stress range and strain amplitude. This implies that the design S-N curve becomes

$$\log_{10} N = -13.164 - 7.912\log_{10}(\frac{S}{2E}\gamma_m) \tag{26}$$

5.4 Calibration

For the same series of values of the section modulus W that was used for the reliability analyses, deterministic structural analyses are carried out. For each value of W, the accumulated damage D is calculated according to Eq. (9) on the basis of values of Δn predicted from the design stress range distribution in conjunction with values of N predicted from the design S-N curve. Pairs of partial safety factors (γ_f, γ_m) are determined in such a way that this accumulated damage becomes exactly equal to the limit value of 1.0 that indicates failure. This is conveniently done by calculating the accumulated damage D for many trial pairs (γ_f, γ_m) and picking those pairs for which D=1.0 results. For each value of W, there will be an infinite number of pairs (γ_f, γ_m) that will lead to D=1.0. This is a result of the form of the limit state function for fatigue failure in flapwise bending and implies an arbitrariness in selecting the partial safety factors (γ_f, γ_m), as the requirement turns out to be on their product. Hence, the result of this exercise performed for many section modulus values W is a required partial safety factor product $\gamma_f\gamma_m$ as a function of the section modulus W, $\gamma_f\gamma_m=\gamma_f\gamma_m(W)$.

The result of the structural reliability analyses, $\beta=\beta(W)$, and the result of the deterministic structural analyses, $\gamma_f\gamma_m=\gamma_f\gamma_m(W)$, are combined by elimination of the section modulus W to give the reliability index β as a function of the calibrated partial safety factor product $\gamma_f\gamma_m$, $\beta=\beta(\gamma_f\gamma_m)$, see Figure 2.

As discussed above, an infinite number of possible choices for the set of partial safety factors (γ_f,γ_m) exist for each β value, as the requirement is on their product. A robust choice of partial safety factors is usually a set which leads to design values of stresses and strengths as close as possible to the design point values resulting from the corresponding reliability analysis. This is so because the design point of the reliability analysis represents the most likely outcome of the governing stochastic variables at failure. In the following, it is outlined how such a particular robust set of partial safety factors can be chosen for the present design problem and thereby remedy the arbitrariness in the result of the calibration.

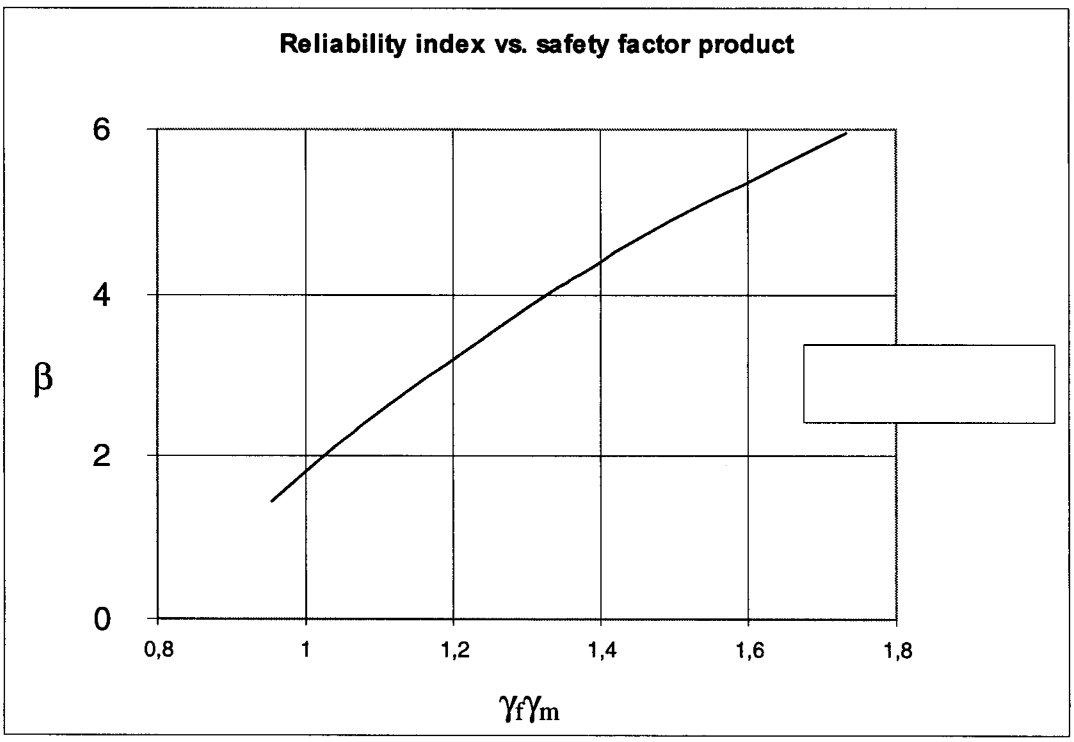

Figure 2 Reliability index β vs. calibrated partial safety factor product $\gamma_f\gamma_m$

Based on the characteristic ε–*N* curve established in Section 6.2 above, the design ε–*N* curve can be expressed as

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10}(\varepsilon\gamma_m) - 2\sigma_e \tag{27}$$

Based on the results of the reliability analysis, the ε–*N* curve in the design point can be expressed as

$$\log_{10} N = \log_{10} K^* - m^* \log_{10} \varepsilon + e_d^* \tag{28}$$

in which $\log_{10}K^*$, m^*, and e_d^* denote design point values of the intercept $\log_{10}K$, the slope m, and the residual e, respectively. The sought-after particular choice for γ_m is achieved by requiring this design point curve to be equal to the design curve and eliminating N. Unfortunately, this will give an expression for γ_m which is not invariant with the strain ε. As an approximation, the design point values for m and $\log_{10}K$ can be replaced by the mean values of these variables. This approximation can be justified by the relatively small uncertainty importance ascribed to these variables as determined by the reliability analysis. Hence, Eq. (28) changes to

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10} \varepsilon + e_d^* \tag{29}$$

and the sought-after particular choice for γ_m becomes

$$\gamma_m = 10^{-\frac{e_d^* + 2\sigma_e}{E[m]}} = 10^{-\frac{e_d^* + 0.792}{7.912}} \tag{30}$$

which is a requirement expressed explicitly in terms of the results of the reliability analysis.

The particular choice for the material factor γ_m in Eq. (30), in conjunction with the requirement to the product $\gamma_f\gamma_m$, is easily seen to suffice for determination of a corresponding particular requirement to the load factor γ_f. The robustness in this particular set of partial safety factors is embedded in the fact that it, by its derivation in compliance with the results of the underlying reliability analysis, reflects appropriately the uncertainty importance information which is a by-product of this analysis. Reference is made to Det Norske Veritas (1992).

As an example, consider a rotor blade design for high safety and less serious consequence, which would be a reasonable classification for a wind turbine design against fatigue where human life is at negligible risk. According to Nordic Committee on Building Regulations (1978), the requirement to the annual failure probability for design under such a classification is 10^{-5}. Under a Poissonian assumption for a rare failure event, this implies that the acceptable failure probability in a 20-year lifetime is $2.0 \cdot 10^{-4}$, and the corresponding target reliability index is β_t=3.54.

For a target reliability index β_t=3.54, Figure 2 gives a requirement to the product of the partial safety factors

$$\gamma_f\gamma_m = 1.252 \tag{31}$$

The underlying reliability analysis is performed for a section modulus W=0.00144 m^3 and yields a design point value e_d*=−1.273 for the ε–N curve residual e. Hence, the requirement to the material factor becomes

$$\gamma_m = 10^{-\frac{e_d^*+0.792}{7.912}} = 10^{-\frac{-1.273+0.792}{7.912}} = 1.150 \qquad (32)$$

and the requirement to the load factor is then implied as

$$\gamma_f = \frac{\gamma_f \gamma_m}{\gamma_m} = 1.088 \qquad (33)$$

The required load factor is found to be fairly close to 1.0, which reflects the fairly small importance attributed to the uncertainty in the loading and the load model as assessed by the reliability analysis. The required material factor is correspondingly found to be 1.15. This may seem not to be a particularly strict requirement. However, the partial safety factors are much dependent on the choices made for the corresponding characteristic values for load and resistance. In the present case, a lower-tail quantile of the resistance properties is chosen as the characteristic resistance. This implies that the characteristic resistance automatically accounts for part of the uncertainty in the resistance, and the material factor is then only meant to account for the remainder of this uncertainty, thus leaving the safety factor requirement in the present case to a mere 1.15.

6. SUMMARY AND CONCLUSIONS

The design of a wind turbine rotor blade against fatigue failure in flapwise bending has been considered. The load history in the design life has been modelled on the basis of observed distributions of bending moments at the blade root of an instrumented prototype rotor blade subjected to wind loads. Statistical uncertainty in the distribution parameters has been estimated and taken into account. The resistance has been modelled in terms of an ε–N curve. Uncertainties in the variables that describe this curve have been estimated and have also been taken into account. The cumulative damage that eventually leads to a fatigue failure has been predicted according to a Miner's sum formulation.

The models for load, resistance, and cumulative damage have been used as a basis for defining a limit state function for fatigue failure, and a first-order reliability analysis of the considered rotor blade against such a failure in flapwise bending has been carried out. The reliability analysis has been interpreted with respect to the probability of failure as well as identification of important uncertainty sources. The inherent variability in the fatigue life as represented by the uncertainty in the residual of the ε–N curve is found to be the single most important uncertainty source.

A reliability-based calibration of partial safety factors for design of the rotor blade against fatigue in flapwise bending has been carried out. A load factor γ_f has been applied to all stresses, and all strengths have been divided by a material factor γ_m. A target lifetime reliability corresponding to an acceptable annual probability of failure of 10^{-5} has been applied for the calibration. Based on a specific choice of characteristic values for load and resistance, a requirement to the product of load factor and material factor $\gamma_f\gamma_m$ =1.252 has come out. Based on the importance information of the underlying reliability analysis, a particular robust set of partial safety factors that fulfill this requirement has been determined, hence γ_f=1.088 and γ_m=1.150.

It is emphasized that the reliability-based safety factor calibration presented herein is site and wind-turbine specific and only applicable to flapwise bending of rotor blades. Different safety factors may result for different sites, different wind turbines, and different blade materials. Similar calibrations can be carried out for such different wind turbines at various sites, and a common set of partial safety factors for a class of wind turbines, sites, and materials can be optimized in dependence of the expected demand for each individual combination of wind turbine, site, and material within the class. Future work is suggested to be devoted to investigations of a series of wind turbines for different sites and blade materials with the ultimate goal of developing a reliability-based optimal design code. Such a code is not to be limited to rotor-blade fatigue in flapwise bending alone, as extensions to other design cases such as rotor-blade fatigue in edgewise bending as well as fatigue of other wind-turbine components are foreseen.

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DNV

TECHNICAL REPORT

Client :	Danish Energy Agency through Risø National Laboratory
Title :	Calibration of Partial Safety Factors for Design of Wind-Turbine Rotor Blades against Fatigue Failure in Flapwise Bending – 300 kW Turbine
Report No. :	97-2050

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REPORT

Date	Dept./Sec.	Project No	Type of Report
October 29, 1997	**DTP342**	**341440**	**Research**

Approved by	Client, Sponsor	Client's ref.
for Det Norske Veritas AS Brian Hayman	**Danish Energy Agency** **through** **Risø National Laboratory**	

Summary

A probabilistic model for evaluation of the safety of a wind-turbine rotor blade against fatigue failure in flapwise bending is presented. The model accounts for uncertainties in load and resistance. The model is applied in conjunction with a first-order reliability method to perform a structural reliability analysis of a particular, site-specific wind turbine. The turbine selected for this purpose is a 300 kW wind turbine. The probability of fatigue failure in flapwise bending of one of the rotor blades of this wind turbine over a twenty-year design life is calculated. It is demonstrated how the reliability analysis results can be used to calibrate partial safety factors for load and resistance for use in conventional deterministic fatigue design.

DNV Rep.No.	Subject Group	4 Indexing terms
97-2050	B3, B4, F1, K0	Structural Reliability Code Calibration Fatigue Wind Turbines

Title of Report

CALIBRATION OF PARTIAL SAFETY FACTORS FOR DESIGN OF WIND-TURBINE ROTOR BLADES AGAINST FATIGUE FAILURE IN FLAPWISE BENDING – 300 KW TURBINE

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Work carried out by	Work verified by
Knut O. Ronold	Jakob Wedel-Heinen

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1. INTRODUCTION

Wind-turbine rotor blades exposed to wind loading are vulnerable to cumulative fatigue damage owing to the cyclic nature of the loading. The wind speed that causes bending of the rotor blades exhibits a natural variability, such that the load amplitudes become random, and the *S–N* curve that gives the number of stress cycles to failure and represents the resistance of the rotor blade material is encumbered with uncertainty owing to a limited number of test specimens as well as variability from one specimen to another.

Partial safety factors are used in structural design as factors on characteristic values of governing load and resistance quantities to account for variabilities and uncertainties in these quantities.

This report demonstrates how a structural reliability method can be applied as a rational means to analyze a wind-turbine rotor blade with respect to fatigue in flapwise bending, and to establish partial safety factors for design of such rotor blades against fatigue failure. A site-specific wind turbine of a prescribed make is considered, here a 300 kW turbine, and probabilistic models for the wind loading and its transfer to bending stresses in the rotor blades is established together with a stochastic representation of the material resistance. The event of fatigue failure in flapwise bending is considered as based on a Miner's sum formulation for cumulative damage.

2. THEORY FOR LOAD, RESISTANCE, AND CUMULATIVE DAMAGE

2.1 Wind Climate and Load History for Rotor Blade

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the turbulence intensity I_T. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \qquad (1)$$

in which k and A are site- and height-dependent coefficients. The turbulence intensity I_T is also site- and height-dependent. It is defined as the standard deviation of the wind speed divided by the mean wind speed U_{10} and represents the gustiness of the wind about this mean. The turbulence intensity I_T is here assumed to be independent of U_{10}, but could in general be modelled as dependent on U_{10} which would be the case for a site with inhomogeneous terrain. Detailed information about the distribution of I_T is not available. The mean value can be taken as

$$E[I_T] = \left(\ln \frac{z}{z_0}\right)^{-1} \qquad (2)$$

where z is the height above the ground, i.e., the hub height of the rotor, and z_0 is the roughness parameter for the terrain. Reference is made to Dansk Ingeniørforening (1992). A representative value of the coefficient of variation is COV=0.25, and the distribution type can be assumed to be lognormal. The (U_{10},I_T) space is discretized into a number of bins ("two-dimensional intervals") of approximately constant values of U_{10} and I_T. The bin width is taken as 2 m/sec for U_{10} and as 0.02 for I_T.

One rotor blade is considered in the following. Let X denote the bending moment range at the blade root in flapwise bending. Hence, X is the double amplitude of the flapwise bending moment response owing to an aerodynamic load cycle that is applied to the rotor blade. One bending moment range is associated with each load cycle, and load cycles are identified by rainflow counting as described by Madsen et al. (1986). Observations of the bending moment range X are recorded in 10-minute intervals. Each 10-minute record of X is binned by U_{10} and I_T. For a particular bin (U_{10},I_T) there will be M 10-minute records of X, and they are used to give an estimate of the long-term distribution of X conditioned on (U_{10},I_T), i.e., $X|(U_{10},I_T)$, on discretized form. The number of load cycles n_{10} in each 10-minute interval is also observed and depends on U_{10} and I_T.

Because the distribution of $X|(U_{10},I_T)$ is encumbered with uncertainty owing to limited data for its estimation, it is desirable to parametrize the distribution and represent this uncertainty in reliability analyses as uncertainty in the distribution parameters. The distribution of $X|(U_{10},I_T)$ can be parametrized in terms of its statistical moments. The first three statistical moments are used for this purpose. These moments are the mean value a_1, the standard deviation a_2, and the skewness a_3. Their expected values $E[a_i]$, i=1,2,3, can be estimated based on the observed discretized version of the conditional distribution of $X|(U_{10},I_T)$. Their standard deviations $D[a_i]$, i=1,2,3, and also their correlation matrix ρ can be estimated by a resampling technique such as the jackknife or the bootstrap, see Efron and Tibshirani (1993).

It is assumed that the expected values and standard deviations of a_1, a_2, and a_3 conditioned on U_{10} and I_T are adequately represented by the polynomial surfaces

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^{\;2} + b_{3i}I_T + b_{4i}I_T^{\;2} \quad (3)$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^{\;2} + c_{3i}I_T + c_{4i}I_T^{\;2} \quad (4)$$

in which the coefficients b_{ji} and c_{ji}, j=0,...4, i=1,2,3, are determined by least-squares regressions of all estimated mean values and standard deviations of a_1, a_2, and a_3 over the (U_{10},I_T) space.

Based on the assumption that the central limit theorem holds for the estimates of the three moments a_1, a_2, and a_3, these moments can be represented as

$$a_i = E[a_i] + U_i D[a_i], \qquad i = 1,2,3 \quad (5)$$

in which $\mathbf{U}=(U_1, U_2, U_3)^T$ is a three-dimensional normally distributed variable with zero mean, unit variance, and correlation matrix ρ. Note in this context that U_i is standard notation for a standard normally distributed variable within the field of structural reliability and is not to be confused with any wind speed. Note also that the vector $\mathbf{U}=(U_1, U_2, U_3)^T$ represents the statistical uncertainty in the three moments a_1, a_2, and a_3 owing to the limited data available for their estimation.

Above, the statistical moments a_1, a_2, and a_3 of the available measured data for the bending moment range X at the blade root in flapwise bending have been dealt with. However, no statement has so far been made with respect to the distribution of the bending moment amplitudes themselves, neither in the short term, conditional on a particular wind climate (U_{10},I_T), nor in the long term such as over the design life of the rotor blade. A model for the distribution of the bending moment range X is therefore dealt with in the following.

Load response amplitudes are often seen to have marginal distributions which are close to Weibull distributions. The bending moment range is two times such a load response amplitude. Based on the first three moments a_1, a_2, and a_3 of the distribution of the conditional bending moment range $X|(U_{10},I_T)$, this distribution can be modelled as a quadratic expansion of a parent Weibull-distributed variable U_W. The parent Weibull-distributed variable U_W is chosen such that it has the same mean a_1 and the same standard deviation a_2 as the distribution of $X|(U_{10},I_T)$ which is to be modelled. For the case that the skewness of U_W is smaller than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a softening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa(U_W + \varepsilon U_W^{\,2}) \tag{6}$$

For the case that the skewness of U_W is greater than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a hardening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa\frac{\sqrt{1+4\varepsilon U_W}-1}{2\varepsilon} \tag{7}$$

In both cases, the model is referred to as a quadratic Weibull model, and the coefficients ε, κ, and $x_{\min}$ are chosen such that the mean value a_1, the standard deviation a_2, and the skewness a_3 of the distribution of $X|(U_{10},I_T)$ are all preserved. The quadratic Weibull model may be thought of as a generalized or distorted Weibull distribution. Reference is made to Lange and Winterstein et al. (1996). Note that the quadratic Weibull distribution provides a better fit to the data, and in particular a better representation of the important upper tail of the distribution, than the distorted lognormal distribution used by Ronold et al. (1994) in a first approach to a parametrized representation of the load range distribution. The distorted lognormal distribution, obtained by a logarithmic Hermite polynomial expansion of a parent Gaussian distribution, is known to have a heavy upper tail which, as commented by Ronold et al. (1994), leads to overprediction of upper quantiles of the bending moment ranges and thereby of the high-range stresses. Note also that the quadratic Weibull model provides results very close to those obtained by a cubic Weibull model which preserves the first four statistical moments of the distribution of $X|(U_{10},I_T)$, including the kurtosis a_4, but which is computationally much more cumbersome and time-consuming and therefore less attractive, see Ronold et al. (1996). The accuracy of predictions made by means of the quadratic Weibull models of Eqs. (6) and (7) is further dealt with later.

The section modulus at the rotor blade at the blade root is W, and the stress range S corresponding to the moment range X is $S=X/W$. When a discretization of the stress range space is introduced, either Eq. (6) or Eq. (7), depending on the value of the skewness a_3, can be applied in conjunction with the distribution function of the parent Weibull variable U_W to calculate the probability content of each interval ΔS of this discretization. The corresponding number of cycles within each such interval in a 10-minute period can be determined as this probability content times the total number of cycles $n_{10}(U_{10},I_T)$.

Integrating contributions from all possible 10-minute wind climate bins (U_{10},I_T), weighted according to the quoted Weibull distribution for U_{10} and the lognormal distribution for I_T, this can be used to establish an ordered history of the stress range S over the design life T_L of the rotor blade. This compound lifetime distribution of the stress range S can be expressed in terms of the number of stress cycles n whose associated stress range exceeds a level S during the design life of the rotor blade, see Figure 1. This, in turn, can be used to calculate the number of stress cycles Δn within an interval ΔS of the discretization of the stress range space.

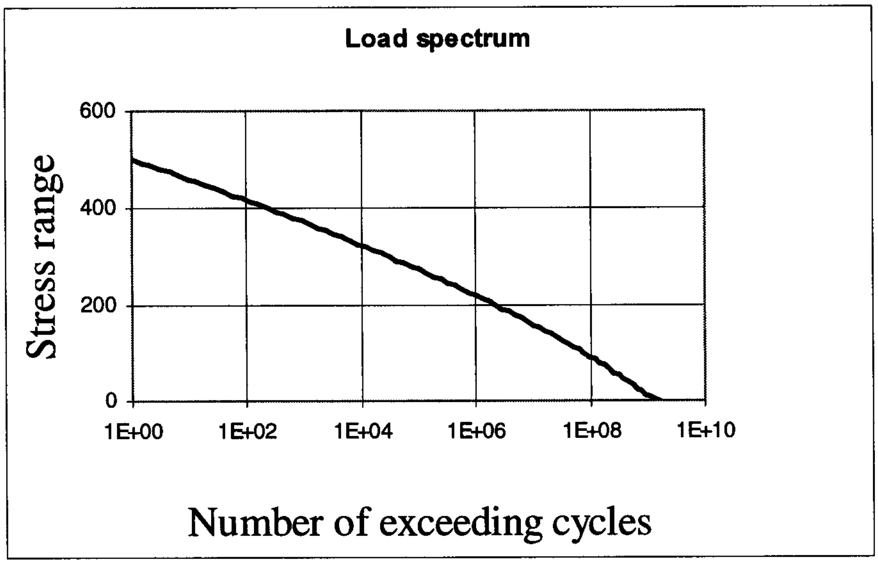

Figure 1 Example of compound stress range distribution over a design life

The number of cycles $n_{10}(U_{10},I_T)$ in a 10-minute interval is also encumbered with uncertainty. However, the coefficient of variation is inversely proportional with the square-root of the interval length, such that when $n_{10}(U_{10},I_T)$ is scaled to give the number of cycles over a long time span such as a 20-year design life, the uncertainty in this number becomes insignificant and can be ignored. In the reliability analysis $n_{10}(U_{10},I_T)$ is therefore left as a deterministic quantity only dependent on U_{10} and I_T.

2.2 Fatigue Strength and *S-N* curve

For a given stress range S, the number of cycles N to failure is generally expressed through an S-N curve, $N=BS^{-k}$. However, in tests of composite materials for use in rotor blades, the strain amplitude ε is usually measured rather than the stress range S. Hence, for such materials the number of cycles N to failure is expressed through an ε–N curve. This curve can be expressed by the following relation

$$\log_{10} N = \log_{10} K - m\log_{10} \varepsilon \tag{8}$$

in which K and m are coefficients. This gives a linear model for $\log_{10}N$

$$\log_{10} N_i = \log_{10} K - m\log_{10} \varepsilon_i + e_i \ , i=1,...n \tag{9}$$

in which the pair ($\log_{10}K$, m) describes the expected behavior and can be estimated by a linear regression analysis based on n observed data pairs (ε_i,N_i). The zero-mean terms e_i denote residuals that represent local variations from test specimen to test specimen, or from one point of the rotor blade to another. The standard deviation σ_e of the residuals e_i will result as a byproduct of the regression analysis, and so will the standard deviations and correlation coefficient of $\log_{10}K$ and m. The stress range S that corresponds to the strain amplitude ε can be expressed as S=2$E\varepsilon$, where E denotes the modulus of elasticity of the material in the direction of the loading. The modulus of elasticity is idealized as a constant here, but may in general vary with the magnitude of the strain. A refined representation with such a variation included would be desirable. Further, the possible effect of a non-zero mean stress has been ignored, mainly because of limitations in available test data. Scale effects from test specimen to prototype and long-term environmental degradation effects owing to exposure to moisture and ultraviolet light have been left out of consideration. Also the inclusion of such effects would be desirable if data would permit.

2.3 Cumulative Damage and Failure Criterion in Fatigue

According to Miner's rule, fatigue failure in a structural material is defined to occur when the accumulated damage D exceeds 1.0, where D is defined as

$$D = \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{10}$$

Here, Δn is the number of load cycles at stress range S in the lifetime of the rotor blade, and N is the number of cycles to failure at this stress range. The sum is over all stress ranges S_i in a sufficiently fine discretization of the stress range space.

3. PROBABILISTIC AND DETERMINISTIC MODELLING

The reliability of a site-specific wind turbine against fatigue failure of one of its rotor blades in flapwise bending is considered. The reliability is computed by a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The input to the reliability analysis consists of a limit state function, specified in terms of a set of basic variables which consist of stochastic variables as well as deterministic parameters. Furthermore, the statistical distributions of the stochastic variables must be given, and the values of the deterministic parameters must be specified. The following sections describe the stochastic variables, the deterministic parameters, and the limit state function. Separate sections are devoted to make and site of wind turbine, environmental loading, fatigue strength, model uncertainty, and limit state function.

3.1 Wind Turbine Characteristics

A 300 kW wind turbine is considered. The hub height is z=30 m, and the section modulus of the rotor blade at the blade root in flapwise bending is taken as W=0.00067 m^3.

3.2 Environmental Loading

The wind turbine is considered for a location whose wind loading regime is characterized by a scale parameter A=9.1 m/sec, a slope parameter k=1.9, and a terrain roughness z_0=0.05 m, the latter thus implying a mean value of the turbulence intensity of $E[I_T]$ =0.156.

A total of 139 10-minute records of flapwise bending moment ranges X for various wind climate realizations (U_{10},I_T), considered fixed within each 10-minute interval, are available. This is a rather limited amount of data, at least when considering the amounts available from other monitored wind turbines, see for example Ronold et al. (1999). For each bin (U_{10},I_T), the available M 10-minute moment range records are merged, and the observed moment ranges X are sorted in increasing order. From this, the cumulative distribution function of $X|(U_{10},I_T)$ is derived, and its first three moments, here denoted a_1 through a_3, are estimated. The standard deviations of these three moments are obtained by jackknifing the M 10-minute records. As stated in a previous section, the following model is chosen to represent the coefficients a_1 through a_3

$$a_i = E[a_i] + U_i D[a_i], \quad i = 1,2,3 \tag{11}$$

in which the mean value $E[a_i]$ and standard deviation $D[a_i]$ of the ith moment a_i are represented as

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^2 + b_{3i}I_T + b_{4i}I_T^2 \tag{12}$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^2 + c_{3i}I_T + c_{4i}I_T^2 \tag{13}$$

Table 1 Estimated Coefficients in Polynomial Model for $E[a_i]$

i	b_0	b_1	b_2	b_3	b_4
1	-10.462	1.4878	-0.0109	124.63	-253.68
2	-5.1029	0.7697	0.0099	92.196	-198.34
3	1.5233	-0.0732	-0.0022	4.6388	-15.416

Table 2 Estimated Coefficients in Polynomial Model for $D[a_i]$

i	c_0	c_1	c_2	c_3	c_4
1	-1.0227	0.0953	-0.0020	8.1962	-10.482
2	-0.0337	0.1048	-0.0027	-10.050	64.273
3	0.4263	-0.0315	0.0014	-3.6612	15.213

The polynomial coefficients in these expressions are determined by a least-squares regression of the available data and are presented in Tables 1 and 2. The stochastic variables denoted $\mathbf{U}=(U_1,U_2,U_3)^T$ represent the statistical uncertainties in the bending moment range distributions and follow a three-dimensional normal distribution with mean values 0.0, standard deviations 1.0, and a correlation matrix which is estimated to be

$$\rho=\begin{bmatrix} 1.000 & 0.899 & 0.006 \\ 0.899 & 1.000 & 0.085 \\ 0.006 & 0.085 & 1.000 \end{bmatrix} \tag{14}$$

The number of aerodynamic stress cycles n_{10} in a 10-minute interval is represented as a function of (U_{10},I_T) as follows

$$n_{10}=597.8+68.52U_{10}-1.341U_{10}^2-904.76I_T+1896I_T^2 \tag{15}$$

in which the coefficients are estimated by a least-squares regression from a total of 139 records of n_{10}, when U_{10} is quoted in units of m/sec.

As described in a previous section, the conditional distribution of $X|(U_{10},I_T)$, expressed in terms of a parent Weibull distribution, is used in conjunction with the long-term distributions of U_{10} and I_T as well as the number of cycles $n_{10}(U_{10},I_T)$ in 10-minute intervals to establish an ordered lifetime history of the bending moment range X for the considered rotor blade in flapwise bending. The corresponding history of the bending stress range S of interest for prediction of cumulative damage by Miner's sum is easily derived by division by the section modulus W, hence $S=X/W$.

3.3 Resistance and Stiffness of Composite Laminate

As stated in a previous section, the ε–N curve that gives the number of stress cycles N to failure as a function of the strain amplitude ε is given by the linear relationship

$$\log_{10} N = \log_{10} K - m\log_{10}\varepsilon + e \tag{16}$$

for the rotor blade laminate.

A total of 81 observed pairs (ε,N) are available from laboratory tests on specimens of a polyester laminate reinforced by five layers of combined woven glass roving and chopped strand mat with fibres oriented at 0/90 during testing and with some fibres in the load direction. A regression analysis of these data pairs according to the linear model leads to the following estimates of mean values, standard deviations, and correlation coefficient for the coefficients $\log_{10} K$ and m

$$E\begin{bmatrix}\log_{10} K \\ m\end{bmatrix}=\begin{bmatrix}-12.372 \\ 7.912\end{bmatrix} \qquad D\begin{bmatrix}\log_{10} K \\ m\end{bmatrix}=\begin{bmatrix}0.513 \\ 0.247\end{bmatrix} \qquad \rho=-0.996 \tag{17}$$

when strain amplitudes are quoted as a dimensionless absolute quantity. Under the central limit theorem the distribution of ($\log_{10}K$,m) is a bivariate normal distribution. The standard deviation of the residual term e is estimated to be σ_e=0.396. The zero-mean residual term e is represented by a normal distribution with this standard deviation. For details about the data and the tests of the laminates, reference is made to Echtermeyer et al. (1993) and Echtermeyer (1994). A constant modulus of elasticity is used, E=29.7·10^6 kPa.

3.4 Model Uncertainty

Model uncertainty can be associated with all simplifications and idealizations made in the formulation of the engineering models that are used for analysis of fatigue damage and failure of a rotor blade in bending. One of these model uncertainties is considered here, namely that which is associated with the use of the quadratic Weibull model for representation of the distribution of the bending moment range conditional on the wind climate (U_{10},I_T). Damage predictions by the Miner sum, based on such quadratic Weibull distributions for the loading, are therefore multiplied by a random factor F_M. This random factor represents the bias and uncertainty in these damage predictions as associated with the use of the quadratic Weibull model for the conditional load distributions. Based on data reported by Ronold et al. (1999) for a similar wind turbine, the following values for the mean value and standard deviation of the random model uncertainty factor F_M are chosen

$$E[F_M]=0.58 \qquad D[F_M]=0.15 \tag{18}$$

The distribution of F_M is taken as a normal distribution.

It appears that a significant bias in the damage predictions by the quadratic Weibull model for the loading is present and accounted for by a mean value of F_M which is considerably less than 1.0. However, because the fatigue damage is very nonlinear with respect to the loading, this bias corresponds only to a mere 5-7% overprediction, on average, of the bending moment ranges by the quadratic Weibull model. This may serve to support use of the quadratic Weibull model as a fairly accurate model for representation of flapwise bending moment ranges of rotor blades, in particular when considering it is based on a fit to the first three statistical moments only.

3.5 Limit State Function

The reliability against fatigue failure of the considered rotor blade in flapwise bending is analyzed for the cyclic loading caused by wind over the design life. For this purpose, a limit state function is defined

$$g(\mathbf{X}) = 1 - F_M D(\mathbf{X}) = 1 - F_M \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{19}$$

in which D is the predicted cumulative fatigue damage expressed through the Miner's sum as defined in a previous section, and $\mathbf{X}$ denotes the vector of stochastic variables which include the variables $\mathbf{U}$ that represent the uncertainty in the loading, the variables ($\log_{10}K$,m,e) that represent the uncertainty in the resistance, and the variable F_M that represents the bias and uncertainty in the cumulative damage predictions as resulting from use of the quadratic Weibull model for the loading.

4. RELIABILITY ANALYSES

The reliability is the complement of the failure probability

$$P_F = P[g(\mathbf{X}) \leq 0] \tag{20}$$

and may be expressed in terms of the reliability index $\beta=-\Phi^{-1}(P_F)$. The reliability is computed by means of a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The probabilistic analysis program PROBAN, see Tvedt (1989), is used for this purpose. The results of the reliability analysis are presented in Table 3.

Table 3 Results of Reliability Analysis
20-Year Lifetime Fatigue in Flapwise Bending
Rotor Blade, W=0.00067 m^3

Probability of Failure $P_F = 1.35 \cdot 10^{-4}$
Reliability Index β=3.64

Variable	Distribution	Design point $\mathbf{x}$*	Importance factor α^2
U_1	Normal	1.0829	0.225
U_2	Normal	1.2161	
U_3	Normal	1.3086	
$\log_{10}K$	Normal	-12.003	0.052
m	Normal	7.726	
e	Normal	−1.1929	0.684
F_M	Normal	0.6888	0.039

By examination of the resulting importance factors reported in the fourth column of Table 3, it appears that the inherent variability in the fatigue life as represented by the uncertainty in the residual e of the ε–N curve is by far the single most important uncertainty source. As much as 68% of the total uncertainty importance is attributed to this resistance variable, while the other ε–N curve variables m and $\log_{10}K$ vouch for another 5% of the uncertainty importance. This leaves 23% uncertainty importance ascribed to the uncertainty in the load variables U_1, U_2, and U_3, and 4% ascribed to the load model uncertainty factor F_M. Note that the 23% uncertainty importance associated with the loading represents a rather significant contribution to the total uncertainty and is a result of significant statistical uncertainty owing to the limited amount of observed load history records.

An inspection of the computational results reveals that the major contribution to the cumulative damage is ascribed to the about 10^7 medium-amplitude stress cycles in a couple or more cycle-number decades centered about $\log_{10}N$=6 in the bending moment range distribution. This is a fairly small fraction of the total of about 10^9 stress cycles that occur over the design life of the rotor blade. The fact that such a low fraction of the applied stress cycles vouches for most of the cumulative damage can be ascribed to the value of the slope parameter m of the ε–N curve which is approximately equal to 8 for the composite laminate in the present case. Stress ranges are raised to the mth power for prediction of the number of cycles to failure. This implies that the higher the value of m, the more dominant are the high stress ranges. A comparison can be made with welded steel details whose m values are usually in the range 3-4 and whose major contribution to accumulated fatigue damage is ascribed to the low-amplitude stress cycles that correspond to the lower right part of the stress range distribution in Figure 1. These stress cycles form the majority of the total number of stress cycles over the design life. An interesting consequence of this dependency of the cumulative damage on the value of m is that if epoxy materials are considered for the rotor blade, for which m values of up to 12 or 13 are seen, the fatigue problem may be turned into an extreme value problem as far as the loading goes. Even for the present m value of 8, this indicates how important a proper estimation of the upper tail of the load distribution is.

5. CALIBRATION OF PARTIAL SAFETY FACTORS

5.1 Philosophy

It is of interest to demonstrate how reliability analysis results, obtained as outlined in the previous chapters, play a role in codified practice and design. Calibration of partial safety factors for design is an important application. With the first-order reliability method available, it is possible to determine sets of equivalent partial safety factors which result in rotor blade designs with a prescribed reliability. As a first step, a target reliability index β_t must be selected.

The choice for the target reliability index can be derived from a utility-based feasibility assessment in a decision analysis, or by requiring that the safety level as resulting from the design by a reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society. A range of target reliability indices will be considered in the following.

In the case of a prescribed reliability level which is different from the one that results from an actually executed reliability analysis of a wind turbine rotor blade, the geometrical quantities of the blade have to be adjusted such that this required reliability level results from a reliability analysis of the modified blade. The geometrical quantities which can be adjusted to achieve a specified reliability level are sometimes denoted design parameters. It is most practicable to operate on just one such design parameter when adjusting the design in order to reach the specified reliability level. For a rotor blade, the most practicable parameter to adjust is the section modulus W which is a function of the cross-sectional properties of the blade at the root.

To cover a range of target reliabilities, a series of reliability analyses is carried out for a range of values for the section modulus W. This gives the reliability index β as a function of the section modulus W, $\beta=\beta(W)$. In addition, as a byproduct of the reliability analyses, the values of the design point e_d* of the ε–N curve residual e prove useful in the following and are retained.

5.2 Characteristic Values of Governing Variables

Characteristic values have to be selected for the governing load and resistance variables. For design in ultimate loading, the 98% quantile of the annual maximum load is traditionally used as the characteristic load value. For design in fatigue loading, this has hardly any meaning, as a characteristic load distribution for the design life needs to be selected rather than such a single characteristic load value. The governing load distribution is a compound distribution of the bending moment range over the design life of the rotor blade and has contributions from many moment range distributions conditioned on different 10-minute wind climates (U_{10},I_T). For simplicity, an idealized characteristic moment range distribution is adopted here,

$$X = k_R X_0 \left(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}\right) \tag{21}$$

in which X denotes the moment range which is exceeded in n stress cycles during the design life T_L, N_r=$f_r T_L$ is the number of rotor cycles in this life, f_r denotes the rotor frequency, X_0 is a characteristic bending moment whose derivation is described below, and k_R is a scaling factor. A value of k_R =2.43 is applied in the following. This value has been calibrated as the one that will lead to a cumulative damage approximately equal to the median damage for the true compound moment range distribution derived from the observed conditional moment range distributions as described above.

In accordance with the Danish code, see Dansk Ingeniørforening (1992), the characteristic bending moment is defined as

$$X_0 = \frac{\rho}{2} w^2 c C_L \frac{R^2}{3} \tag{22}$$

in which R is the length or radius of the rotor blade measured from the center of the rotor to the tip, c is characteristic chord length at 2R/3, C_L=1.5 is a lift coefficient at 2R/3, ρ=1.28 kg/m^3 is the density of air, and w is a reference wind speed defined by

$$w^2 = \left(\frac{4\pi}{3} f_r R\right)^2 + v_0^{\,2} \tag{23}$$

where f_r is the rotor frequency as before, and v_0 is the 10-minute mean wind speed at stalling of the entire rotor blade. For the considered 300 kW wind turbine, the following numbers apply: T_L=20 years, f_r =0.5167 sec^{-1}, N_r =0.326·10^9 rotations, R=15.5 m, c=1.07 m, and v_0=14.6 m/sec.

For ε–N curves, it is a standard approach to select the characteristic ε–N curve as the curve that results when the estimated ε–N curve is shifted to the left by a distance equal to two times the standard deviation of the residual e, hence

$$\begin{aligned}\log_{10} N &= E[\log_{10} K] - E[m]\log_{10}\varepsilon - 2\sigma_e \\ &= -12.372 - 7.912\log_{10}\varepsilon - 2\cdot 0.396 \\ &= -13.164 - 7.912\log_{10}\varepsilon\end{aligned} \tag{24}$$

when values for the rotor blade laminate are substituted. Reference is made to Det Norske Veritas (1984) and Dansk Ingeniørforening (1992).

5.3 Partial Safety Factors and Design Load and Resistance Properties

Two partial safety factors are introduced. A load factor γ_f greater than 1.0 is applied as a factor on all load values of the characteristic moment range distribution and the relation $S=X/W$ between stress range S and moment range X is substituted such that the design stress range distribution becomes

$$S = \gamma_f k_R \frac{X_0}{W}(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}) \tag{25}$$

Correspondingly, γ_m is a material factor greater than 1.0. For any given number of cycles to failure, the characteristic strength is divided by this number to give the design strength. Recall the relationship $S=2E\varepsilon$ between stress range and strain amplitude. This implies that the design S-N curve becomes

$$\log_{10} N = -13.164 - 7.912\log_{10}(\frac{S}{2E}\gamma_m) \tag{26}$$

5.4 Calibration

For the same series of values of the section modulus W that was used for the reliability analyses, deterministic structural analyses are carried out. For each value of W, the accumulated damage D is calculated according to Eq. (9) on the basis of values of Δn predicted from the design stress range distribution in conjunction with values of N predicted from the design S-N curve. Pairs of partial safety factors (γ_f,γ_m) are determined in such a way that this accumulated damage becomes exactly equal to the limit value of 1.0 that indicates failure. This is conveniently done by calculating the accumulated damage D for many trial pairs (γ_f,γ_m) and picking those pairs for which D=1.0 results. For each value of W, there will be an infinite number of pairs (γ_f,γ_m) that will lead to D=1.0. This is a result of the form of the limit state function for fatigue failure in flapwise bending and implies an arbitrariness in selecting the partial safety factors (γ_f,γ_m), as the requirement turns out to be on their product. Hence, the result of this exercise performed for many section modulus values W is a required partial safety factor product $\gamma_f\gamma_m$ as a function of the section modulus W, $\gamma_f\gamma_m=\gamma_f\gamma_m(W)$.

The result of the structural reliability analyses, $\beta=\beta(W)$, and the result of the deterministic structural analyses, $\gamma_f\gamma_m=\gamma_f\gamma_m(W)$, are combined by elimination of the section modulus W to give the reliability index β as a function of the calibrated partial safety factor product $\gamma_f\gamma_m$, $\beta=\beta(\gamma_f\gamma_m)$, see Figure 2.

As discussed above, an infinite number of possible choices for the set of partial safety factors (γ_f,γ_m) exist for each β value, as the requirement is on their product. A robust choice of partial safety factors is usually a set which leads to design values of stresses and strengths as close as possible to the design point values resulting from the corresponding reliability analysis. This is so because the design point of the reliability analysis represents the most likely outcome of the governing stochastic variables at failure. In the following, it is outlined how such a particular robust set of partial safety factors can be chosen for the present design problem and thereby remedy the arbitrariness in the result of the calibration.

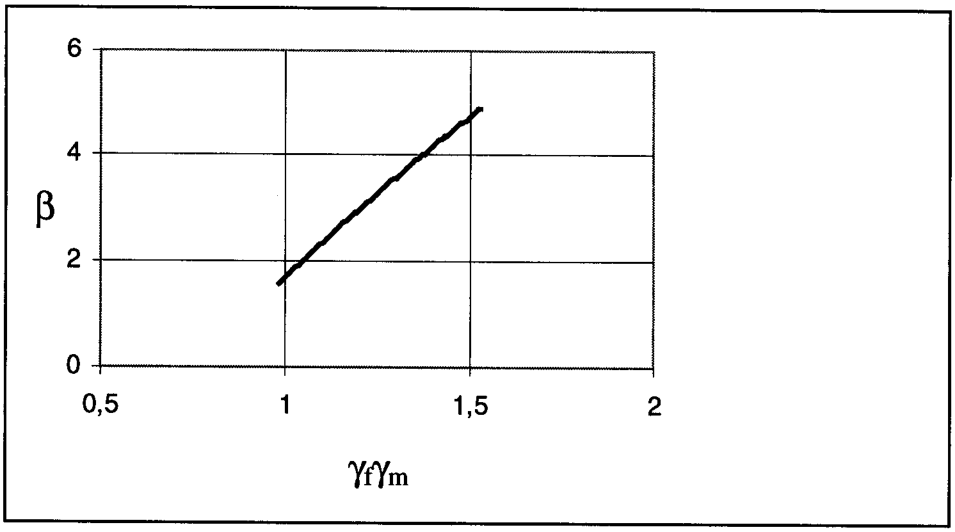

Figure 2 Reliability index β vs. calibrated partial safety factor product $\gamma_f\gamma_m$

Based on the characteristic ε–N curve established in Section 6.2 above, the design ε–N curve can be expressed as

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10}(\varepsilon\gamma_m) - 2\sigma_e \tag{27}$$

Based on the results of the reliability analysis, the ε–N curve in the design point can be expressed as

$$\log_{10} N = \log_{10} K^* - m^* \log_{10} \varepsilon + e_d^* \tag{28}$$

in which $\log_{10}K^*$, m^*, and e_d^* denote design point values of the intercept $\log_{10}K$, the slope m, and the residual e, respectively. The sought-after particular choice for γ_m is achieved by requiring this design point curve to be equal to the design curve and eliminating N. Unfortunately, this will give an expression for γ_m which is not invariant with the strain ε. As an approximation, the design point values for m and $\log_{10}K$ can be replaced by the mean values of these variables. This approximation can be justified by the relatively small uncertainty importance ascribed to these variables as determined by the reliability analysis. Hence, Eq. (28) changes to

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10} \varepsilon + e_d^* \tag{29}$$

and the sought-after particular choice for γ_m becomes

$$\gamma_m = 10^{-\frac{e_d^* + 2\sigma_e}{E[m]}} = 10^{-\frac{e_d^* + 0.792}{7.912}} \tag{30}$$

which is a requirement expressed explicitly in terms of the results of the reliability analysis.

The particular choice for the material factor γ_m in Eq. (30), in conjunction with the requirement to the product $\gamma_f\gamma_m$, is easily seen to suffice for determination of a corresponding particular requirement to the load factor γ_f. The robustness in this particular set of partial safety factors is embedded in the fact that it, by its derivation in compliance with the results of the underlying reliability analysis, reflects appropriately the uncertainty importance information which is a by-product of this analysis. Reference is made to Det Norske Veritas (1992).

As an example, consider a rotor blade design for high safety and less serious consequence, which would be a reasonable classification for a wind turbine design against fatigue where human life is at negligible risk. According to Nordic Committee on Building Regulations (1978), the requirement to the annual failure probability for design under such a classification is 10^{-5}. Under a Poissonian assumption for a rare failure event, this implies that the acceptable failure probability in a 20-year lifetime is $2.0 \cdot 10^{-4}$, and the corresponding target reliability index is β_t=3.54.

For a target reliability index β_t=3.54, Figure 2 gives a requirement to the product of the partial safety factors

$$\gamma_f\gamma_m = 1.296 \tag{31}$$

The underlying reliability analysis is performed for a section modulus W=0.000661 m^3 and yields a design point value e_d^*=−1.161 for the ε–N curve residual e. Hence, the requirement to the material factor becomes

$$\gamma_m = 10^{-\frac{e_d^* + 0.792}{7.912}} = 10^{-\frac{-1.161 + 0.792}{7.912}} = 1.113 \tag{32}$$

and the requirement to the load factor is then implied as

$$\gamma_f = \frac{\gamma_f \gamma_m}{\gamma_m} = 1.164 \tag{33}$$

The required load factor is found to be 1.164, which reflects the importance attributed to the significant statistical uncertainty in the loading and the load model as assessed by the reliability analysis. The required material factor is found to be 1.113. This may not seem to be a particularly strict requirement when considering that the majority of the uncertainty importance is associated with the inherent variability and statistical uncertainty in the ε–N curve. However, the partial safety factors are much dependent on the choices made for the corresponding characteristic values for load and resistance. In the present case, a lower-tail quantile of the resistance properties is chosen as the characteristic resistance. This implies that the characteristic resistance automatically accounts for part of the uncertainty in the resistance, and the material factor is then only meant to account for the remainder of this uncertainty, thus – in the present case – requiring the safety factor for the resistance to equal a value which is somewhat smaller than the requirement to the partial safety factor for the load history.

6. SUMMARY AND CONCLUSIONS

The design of a wind turbine rotor blade against fatigue failure in flapwise bending has been considered. The load history in the design life has been modelled on the basis of observed distributions of bending moments at the blade root of an instrumented prototype rotor blade subjected to wind loads. Measurements from a monitored 300 kW wind turbine with 15.5 m long rotor blades have been used for this purpose. Statistical uncertainty in the distribution parameters has been estimated and taken into account. The resistance has been modelled in terms of an ε–N curve. Uncertainties in the variables that describe this curve have been estimated and have also been taken into account. The cumulative damage that eventually leads to a fatigue failure has been predicted according to a Miner's sum formulation.

The models for load, resistance, and cumulative damage have been used as a basis for defining a limit state function for fatigue failure, and a first-order reliability analysis of the considered rotor blade against such a failure in flapwise bending has been carried out. The reliability analysis has been interpreted with respect to the probability of failure as well as identification of important uncertainty sources. The inherent variability in the fatigue life as represented by the uncertainty in the residual of the ε–N curve is found to be the single most important uncertainty source.

A reliability-based calibration of partial safety factors for design of the rotor blade against fatigue in flapwise bending has been carried out. A load factor γ_f has been applied to all stresses, and all strengths have been divided by a material factor γ_m. A target lifetime reliability corresponding to an acceptable annual probability of failure of 10^{-5} has been applied for the calibration. Based on a specific choice of characteristic values for load and resistance, a requirement to the product of load factor and material factor $\gamma_f\gamma_m$=1.296 has come out. Based on the importance information of the underlying reliability analysis, a particular robust set of partial safety factors that fulfill this requirement has been determined, hence γ_f=1.164 and γ_m=1.113.

It is emphasized that the reliability-based safety factor calibration presented herein is site and wind-turbine specific and only applicable to flapwise bending of rotor blades. Different safety factors may result for different sites, different wind turbines, and different blade materials. Similar calibrations can be carried out for such different wind turbines at various sites, and a common set of partial safety factors for a class of wind turbines, sites, and materials can be optimized in dependence of the expected demand for each individual combination of wind turbine, site, and material within the class. Future work is suggested to be devoted to investigations of a series of wind turbines for different sites and blade materials with the ultimate goal of developing a reliability-based optimal design code. Such a code is not to be limited to rotor-blade fatigue in flapwise bending alone, as extensions to other design cases such as rotor-blade fatigue in edgewise bending as well as fatigue of other wind-turbine components are foreseen.

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TECHNICAL REPORT

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REPORT

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Summary

A probabilistic model for evaluation of the safety of a wind-turbine rotor blade against fatigue failure in edgewise bending is presented. The model accounts for uncertainty and variability in load and resistance. The model is applied in conjunction with a first-order reliability method to perform a structural reliability analysis of one of the rotor blades in a particular, site-specific wind turbine. The turbine selected for this purpose is a 600 kW turbine. The probability of fatigue failure in edgewise bending of one of the rotor blades of this wind turbine over a twenty-year design life is calculated. It is demonstrated how the reliability analysis results can be used to calibrate partial safety factors for load and resistance for use in conventional deterministic fatigue design.

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99-3511	B3, B4, F1, K0

Title of Report

CALIBRATION OF PARTIAL SAFETY FACTORS FOR DESIGN OF WIND-TURBINE ROTOR BLADES AGAINST FATIGUE FAILURE IN EDGEWISE BENDING

4 Indexing terms

- Structural Reliability
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1. INTRODUCTION

Wind-turbine rotor blades exposed to wind and gravity loading are vulnerable to cumulative fatigue damage owing to the cyclic nature of the loading. The wind speed that causes bending of the rotor blades exhibits a natural variability, such that the load amplitudes become random amplitudes. The *S–N* curve that gives the number of stress cycles to failure and represents the resistance of the rotor blade material is encumbered with uncertainty owing to a limited number of test specimens as well as variability from one specimen to another.

Partial safety factors are used in structural design as factors on characteristic values of governing load and resistance quantities to account for variability and uncertainties in these quantities.

This report demonstrates how a structural reliability method can be applied as a rational means to analyze a wind-turbine rotor blade with respect to fatigue in edgewise bending, and to establish partial safety factors for design of such rotor blades against fatigue failure. A site-specific wind turbine of a prescribed make is considered, here a 600 kW turbine with 21.5 m rotor radius, and probabilistic models for the wind loading and its transfer to bending stresses are established together with a stochastic representation of the material resistance. The event of fatigue failure in edgewise bending is considered as based on a Miner's sum formulation for cumulative damage.

2. THEORY FOR LOAD, RESISTANCE, AND CUMULATIVE DAMAGE

2.1 Wind Climate and Load Distribution for Rotor Blade

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the turbulence intensity I_T. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \tag{1}$$

in which k and A are site- and height-dependent coefficients.

The standard deviation σ_U of the wind speed depends on the 10-minute mean wind speed U_{10}. The distribution of σ_U conditioned on U_{10} can be well represented by a lognormal distribution

$$F_{\sigma_U|U_{10}}(s) = \Phi(\frac{\ln s - h_0}{h_1}) \tag{2}$$

in which $\Phi()$ denotes the standard Gaussian cumulative distribution function, and the coefficients h_0 and h_1 depend on U_{10} as follows

$$h_0=k_0+k_1\ln U_{10}$$
$$h_1=k_2+k_3\exp(-k_4 U_{10}) \tag{3}$$

Reference is made to Ronold and Larsen (1999). The turbulence intensity I_T is defined as the standard deviation of the wind speed divided by the mean wind speed U_{10} and represents the gustiness of the wind about this mean, hence $I_T=\sigma_U/U_{10}$. The (U_{10},I_T) space is referred to in the following. This space is discretized into a number of bins of approximately constant values of U_{10} and I_T.

One rotor blade is considered in the following. Let X denote the bending moment range at the blade root in edgewise bending. One bending moment range is associated with each load cycle, and load cycles are identified by rain-flow counting. By means of the rain-flow counting technique, observed bending moment histories, each of 10-minute duration, are converted to 10-minute records of the bending moment range X on histogram form. Within each recorded 10-minute interval, also the wind climate parameters U_{10} and I_T are recorded. The (U_{10},I_T) space is discretized in a number of bins as described before. All 10-minute records of X are sorted according to U_{10} and I_T, i.e., each record is localized to a particular bin (U_{10},I_T) in the adopted discretization of the (U_{10},I_T) space. For a particular bin (U_{10},I_T) there will be a total of M 10-minute records of X, and M varies from bin to bin. These M records are used to give an estimate of the short-term distribution of X conditioned on (U_{10},I_T), i.e., $X|(U_{10},I_T)$, on discretized form. The number of load cycles n_{10} in each 10-minute interval is also observed and depends on U_{10} and I_T.

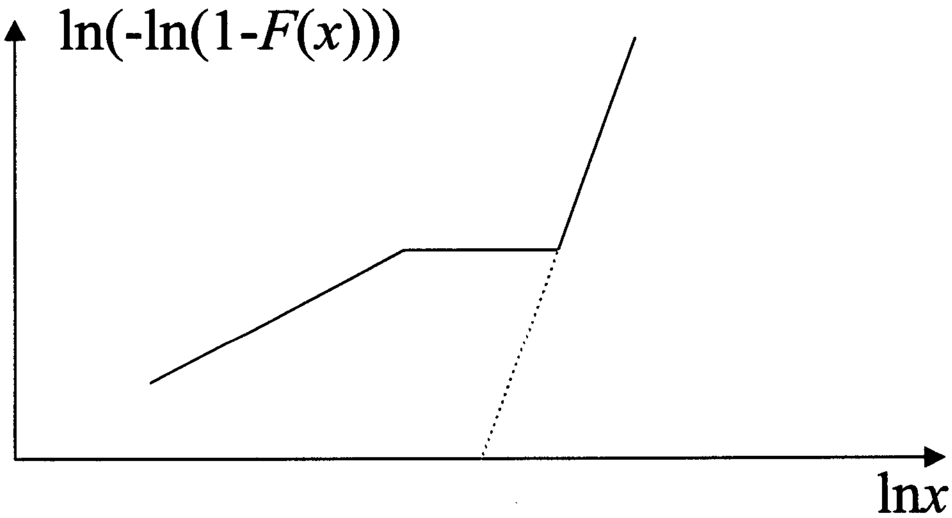

Figure 1 Weibull plot of distribution of edgewise bending moment range conditional on wave climate, $X|(U_{10},I_T)$

The distribution of $X|(U_{10},I_T)$ is bimodal owing to effects of gravity. On Weibull probability paper, the distribution appears to be piecewise linear as indicated in Figure 1, i.e., it is reasonably well represented by two Weibull distributions which are combined. For practical purposes in the following, only the rightmost Weibull line is modelled, extended as indicated in Figure 1. This is a conservative simplification. Because the distribution of $X|(U_{10},I_T)$ is encumbered with uncertainty owing to limited data for its estimation, it is desirable to parametrize the distribution and represent this uncertainty in reliability analyses as uncertainty in the distribution parameters. The Weibull distribution model chosen for representation of the distribution of $X|(U_{10},I_T)$ can be parametrized in terms of the two coefficients of the rightmost Weibull line in Figure 1. The line is given as

$$\ln(-\ln(1-F(x)))=a_0+a_1\ln x \tag{4}$$

in which $F(x)$ denotes the cumulative distribution function for $X|(U_{10},I_T)$. Estimates $\hat{a}_0$ and $\hat{a}_1$ of a_0 and a_1, standard errors in these estimates se($\hat{a}_0$) and se($\hat{a}_1$), and correlation coefficient ρ between the estimates can be found by regression analysis of available data. In general, there are very few data points (often 3 or 4, always less than 20) available for this estimation, dependent on the steepness of the Weibull line and the discretization of the bending moment range axis. As an alternative, the Weibull distribution could have been represented in terms of its mean value and standard deviation. It proves difficult to fit analytical functions to express $\hat{a}_0$, $\hat{a}_1$, se($\hat{a}_0$), and se($\hat{a}_1$) versus (U_{10},I_T). The major reason for this is the very steep Weibull lines which are prevalent in some wind climate regions (U_{10},I_T). The expected coefficient estimates $\hat{a}_0$ and $\hat{a}_1$ are transformed to mean value μ and standard deviation σ through the following relations

$$\mu=(\exp\hat{a}_0)^{-1/\hat{a}_1}\Gamma(1+\frac{1}{\hat{a}_1}) \tag{5}$$

$$\sigma=(\exp\hat{a}_0)^{-1/\hat{a}_1}\sqrt{\Gamma(1+\frac{2}{\hat{a}_1})-\Gamma(1+\frac{1}{\hat{a}_1})^2} \tag{6}$$

in which $\Gamma()$ denotes the gamma function. The mean value μ is rather dominated by the gravitational effects and does only exhibit a limited variation with (U_{10},I_T). The standard deviation σ exhibits more dependency on (U_{10},I_T), yet this dependency is also relatively moderate. It is possible to represent μ and σ by analytical functions of U_{10} and I_T, and the following functions are found to provide adequate models

$$\mu=b_1+b_2U_{10}+b_3U_{10}^2+b_4I_T+b_5I_T^2 \tag{7}$$

$$\sigma=c_1+c_2U_{10}+c_3U_{10}^2+c_4I_T^{c5} \tag{8}$$

in which the coefficients b_j and c_j, j=1,...5, are determined by least-squares regressions of all estimated mean values μ and standard deviations σ over the discretized (U_{10},I_T) space. Values for μ and σ estimated from these functions are transformed to values of $\hat{a}_0$ and $\hat{a}_1$ for use in calculations.

Analytical functions for the coefficients of variation are also fitted

$$\frac{se(\hat{a}_0)}{\hat{a}_0} = d_1+d_2U_{10}+d_3U_{10}^2+d_4I_T+d_5I_T^2 \tag{9}$$

$$\frac{se(\hat{a}_1)}{\hat{a}_1} = e_1+e_2U_{10}+e_3U_{10}^2+e_4I_T+e_5I_T^2 \tag{10}$$

in which the coefficients d_j and e_j, j=1,...5, are determined by least-squares regressions as above.

Under the central limit theorem, the coefficients a_0 and a_1 can be represented as

$$a_i = \hat{a}_i + U_i \cdot se(\hat{a}_i), \quad i = 0,1 \tag{11}$$

in which $\mathbf{U}=(U_0, U_1)^T$ is a bivariate normally distributed variable with zero mean, unit variance, and correlation coefficient ρ. Note in this context that U_i is standard notation for a standard normally distributed variable within the field of structural reliability and is not to be confused with any wind speed. Note also that the vector $\mathbf{U}=(U_0, U_1)^T$ represents the statistical uncertainty in the Weibull coefficients a_0 and a_1 owing to the limited data available for their estimation.

The section modulus at the rotor blade at the blade root is W, and the stress range S corresponding to the moment range X is $S=X/W$. When a discretization of the stress range space is introduced, Eq. (4) can be applied to calculate the probability content of each interval ΔS of this discretization. The corresponding number of cycles within each such interval in a 10-minute period can be determined as this probability content times the total number of cycles $n_{10}(U_{10},I_T)$.

Integrating contributions from all possible 10-minute wind climate bins (U_{10},I_T), weighted according to the quoted Weibull distribution for U_{10} and the lognormal distribution for I_T, this can be used to establish a distribution of the stress range S over the design life T_L of the rotor blade. This compound lifetime distribution of the stress range S can be expressed in terms of the number of stress cycles n whose associated stress range exceeds a level S during the design life of the rotor blade. An example of such a lifetime distribution is given later. The compound lifetime distribution of stress ranges can, in turn, be used to calculate the number of stress cycles Δn within an interval ΔS of the discretization of the stress range space.

The number of cycles $n_{10}(U_{10},I_T)$ in a 10-minute interval is also encumbered with uncertainty. However, the coefficient of variation is inversely proportional with the square-root of the interval length, such that when $n_{10}(U_{10},I_T)$ is scaled to give the number of cycles over a long time span such as a 20-year design life, the uncertainty in this number becomes insignificant and can be ignored. In the reliability analysis $n_{10}(U_{10},I_T)$ is therefore left as a deterministic quantity only dependent on U_{10} and I_T.

2.2 Fatigue Strength and *S–N* curve

For a given stress range S, the number of cycles N to failure is generally expressed through an S–N curve, $N=BS^{-k}$. However, in tests of composite materials for use in rotor blades, the strain amplitude ε is usually measured rather than the stress range S. Hence, for such materials the number of cycles N to failure is expressed through an ε–N curve. This curve can be expressed by the following relationship

$$\log_{10} N = \log_{10} K - m\log_{10} \varepsilon \tag{12}$$

in which K and m are coefficients. This gives a linear model for $\log_{10}N$

$$\log_{10} N_i = \log_{10} K - m\log_{10} \varepsilon_i + e_i \ , i\text{=}1,\ldots n \tag{13}$$

in which the pair ($\log_{10}K$, m) describes the expected behavior and can be estimated by a linear regression analysis based on n observed data pairs (ε_i,N_i). The zero-mean terms e_i denote residuals that represent local variations from test specimen to test specimen, or from one point of the rotor blade to another. The standard deviation σ_e of the residuals e_i will result as a byproduct of the regression analysis, and so will the standard deviations and correlation coefficient of $\log_{10}K$ and m. The stress range S that corresponds to the strain amplitude ε can be expressed as S=2$E\varepsilon$, where E denotes the modulus of elasticity of the material in the direction of the loading. The modulus of elasticity is idealized as a constant here, but may in general vary with the magnitude of the strain. A refined representation with such a variation included would be desirable. Further, the possible effect of a non-zero mean stress has been ignored, mainly because of limitations in available test data. Scale effects from test specimen to prototype and long-term environmental degradation effects owing to exposure to moisture and ultraviolet light have been left out of consideration. Also the inclusion of such effects would be desirable if data would permit.

2.3 Cumulative Damage and Failure Criterion in Fatigue

According to Miner's rule, fatigue failure in a structural material is defined to occur when the accumulated damage D exceeds 1.0, where D is defined as

$$D = \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{14}$$

Here, Δn is the number of load cycles at stress range S in the lifetime of the rotor blade, and N is the number of cycles to failure at this stress range. The sum is over all stress ranges S_i in a sufficiently fine discretization of the stress range space.

3. PROBABILISTIC AND DETERMINISTIC MODELLING

The reliability of a site-specific wind turbine, against fatigue failure of one of its rotor blades in edgewise bending, is considered. The reliability is computed by a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The input to the reliability analysis consists of a limit state function, specified in terms of a set of basic variables, which consist of stochastic variables as well as deterministic parameters. Furthermore, the statistical distributions of the stochastic variables must be given, and the values of the deterministic parameters must be specified. The following sections describe the stochastic variables, the deterministic parameters, and the limit state function. Separate sections are devoted to make and site of wind turbine, environmental loading, fatigue strength, model uncertainty, and limit state function.

3.1 Wind Turbine Characteristics

A 600 kW wind turbine with rotor radius R=21.5 m and hub height z=44 m is considered. The section modulus of the rotor blade at the blade root in edgewise bending is taken as W=0.0030 m^3.

3.2 Environmental Loading

The wind turbine is considered for a location whose wind loading regime is characterized by a scale parameter A=9.1 m/sec and a shape parameter k=1.9 in the long-term Weibull distribution of the 10-minute mean wind speed U_{10}.

As described in a previous section, the distribution of the standard deviation σ_U of the wind speed, conditioned on the 10-minute mean wind speed U_{10}, can be represented by a lognormal distribution

$$F_{\sigma_U|U_{10}}(s) = \Phi(\frac{\ln s - h_0}{h_1}) \tag{15}$$

Based on the available wind climate data from the considered location, the coefficients h_0 and h_1 are represented as functions of U_{10} as follows

$$\begin{aligned} h_0 &= -2.1601 + 1.0326 \cdot \ln U_{10} \\ h_1 &= 0.0579 + 0.6169 \cdot \exp(-0.1709 U_{10}) \end{aligned} \tag{16}$$

A total of 640 10-minute records of edgewise bending moment ranges X for various wind climate realizations (U_{10},I_T), considered fixed within each 10-minute interval, are available. For each bin (U_{10},I_T), the available M 10-minute moment range records are merged, and the observed moment ranges X are sorted in increasing order. From this, the cumulative distribution function of $X|(U_{10},I_T)$ is derived, and its Weibull coefficients, here denoted a_0 and a_1, are estimated by regression of the data points which are judged to belong to the rightmost linear part of the distribution on Weibull probability paper. The standard deviations and the correlation coefficient of the two coefficient estimates are obtained as a byproduct of the regression. As stated in a previous section, the following model is chosen to represent the coefficients a_0 and a_1

$$a_i = \hat{a}_i + U_i \cdot se(\hat{a}_i), \quad i = 0,1 \tag{17}$$

in which the mean values $\hat{a}_i$ and standard deviations $se[\hat{a}_i]$ are represented as described in a previous section through relationships with the mean value μ and standard deviation σ of the Weibull distribution. The following analytical functions are used:

$$\mu = b_1 + b_2 U_{10} + b_3 U_{10}^2 + b_4 I_T + b_5 I_T^2 \tag{18}$$

$$\sigma = c_1 + c_2 U_{10} + c_3 U_{10}^2 + c_4 I_T^{c5} \tag{19}$$

$$\text{CoV}(\hat{a}_0) = \frac{se(\hat{a}_0)}{\hat{a}_0} = d_1 + d_2 U_{10} + d_3 U_{10}^2 + d_4 I_T + d_5 I_T^2 \tag{20}$$

$$\text{CoV}(\hat{a}_1) = \frac{se(\hat{a}_1)}{\hat{a}_1} = e_1 + e_2 U_{10} + e_3 U_{10}^2 + e_4 I_T + e_5 I_T^2 \tag{21}$$

Table 1 Estimated Coefficients in Analytical Models for μ, σ, CoV(â₁), and CoV(â₂)

Quantity	*Coefficient 1*	*Coefficient 2*	*Coefficient 3*	*Coefficient 4*	*Coefficient 5*
μ	310.37	−2.0350	−0.1018	−793.35	2329.2
σ	0.0	−2.1096	0.2661	410.74	1.4344
CoV($\hat{a}_1$)	−0.08545	0.0043	$0.1385 \cdot 10^{-5}$	−0.8026	4.2566
CoV($\hat{a}_2$)	0.08931	−0.0040	$-0.2860 \cdot 10^{-4}$	0.7430	−4.1006

The coefficients in these expressions are determined by a least-squares regression of the available data and are presented in Table 1. The stochastic variables denoted $\mathbf{U}=(U_0,U_1)^T$ represent the statistical uncertainties in the bending moment range distributions and follow a bivariate normal distribution with mean values 0.0, standard deviations 1.0, and a correlation coefficient $\rho=-0.99971$. The latter practically indicates a full negative correlation.

A discussion of the statistical uncertainty associated with measured edgewise bending moment ranges relative to that associated with measured edgewise bending moments is appropriate here.

When the edgewise bending moment range distribution is to be represented, the entire observed distribution is used as a basis, i.e. all observations of the bending moment range are included, see Ronold (1997). This implies that, given the discretization of the bending moment axis, there are usually many data points available for fitting the parameters of a generic distribution. The accuracy is then more dependent on how many 10-minute series of load data are available than on how many data points are included as given by the discretization. In general, there are more 10-minute series available from the frequently occurring low-speed wind climates than from the less frequent high-speed wind climates. Consequently, a larger statistical uncertainty is found for wind climate bins in the high-speed range than for wind climate bins in the low-speed range.

When the edgewise bending moment range distribution is to be represented, only the upper tail of the observed distribution is used as described above, i.e., the rightmost Weibull line in a

Weibull paper plot. Usually there are only a few data points available in such a plot to fit the parameters of this Weibull line. Regardless of how many 10-minute series of load data are available within a considered wind climate bin, the accuracy is governed by the number of data points provided by the discretization of the bending moment axis to describe the upper tail of the distribution. Only a finer discretization of the bending moment axis can remedy this. The steeper the associated Weibull line in a Weibull paper plot is, the fewer are the data points available for fitting the parameters of the Weibull distribution, and the less accurate are the associated parameter estimates. In general, the Weibull line is steeper for low-speed wind climates than for high-speed wind climates. Consequently, a larger statistical uncertainty is found in wind climate bins in the low-speed range than for wind climate bins in the high-speed range. This is the opposite of what is the case for the representation of the flapwise bending moment range distribution, see Ronold (1997).

The number of stress cycles n_{10} in a 10-minute interval is represented as a function of (U_{10},I_T) as follows

$$n_{10}=288.0+234.46U_{10}^{2}-6.510U_{10}^{3}-6013.1I_T^{2}+26733I_T^{3} \tag{22}$$

in which the coefficients are estimated by a least-squares regression from a total of 640 records of n_{10}, when U_{10} is quoted in units of m/sec, and when the limit for U_{10}=0.0 and I_T=0.0 is prescribed to be equal to the theoretical value, which is the number of rotations in 10 minutes.

A design life of T_L=20 years is considered. As described in a previous section, the conditional distribution of $X|(U_{10},I_T)$ is used in conjunction with the long-term distributions of U_{10} and I_T as well as the number of cycles $n_{10}(U_{10},I_T)$ in 10-minute intervals to establish a compound lifetime distribution of the bending moment range X for the considered rotor blade in edgewise bending. The corresponding distribution of the bending stress range S of interest for prediction of cumulative damage by Miner's sum is easily derived by division by the section modulus W, hence $S=X/W$.

3.3 Resistance and Stiffness of Composite Laminate

As stated in a previous section, the ε–N curve that gives the number of stress cycles N to failure as a function of the strain amplitude ε is given by the linear relationship

$$\log_{10} N = \log_{10} K - m\log_{10}\varepsilon + e \tag{23}$$

for the rotor blade laminate.

A total of 81 observed pairs (ε,N) are available from laboratory tests on specimens of a polyester laminate reinforced by five layers of combined woven glass roving and chopped strand mat with fibres oriented at 0/90 during testing and with some fibres in the load direction. A regression analysis of these data pairs according to the linear model leads to the following estimates of mean values, standard deviations, and correlation coefficient for the coefficients $\log_{10} K$ and m

$$E\begin{bmatrix}\log_{10} K \\ m\end{bmatrix} = \begin{bmatrix}-12.372 \\ 7.912\end{bmatrix} \qquad D\begin{bmatrix}\log_{10} K \\ m\end{bmatrix} = \begin{bmatrix}0.513 \\ 0.247\end{bmatrix} \qquad \rho = -0.996 \tag{24}$$

when strain amplitudes are quoted as a dimensionless absolute quantity. Under the central limit theorem the distribution of ($\log_{10}K$,m) is a bivariate normal distribution. The standard deviation of the residual term e is estimated to be σ_e=0.396. The zero-mean residual term e is represented by a normal distribution with this standard deviation. For details about the data and the tests of the laminates, reference is made to Echtermeyer et al. (1993) and Echtermeyer (1994).

A constant modulus of elasticity is used, E=29.7·10^6 kPa.

3.4 Limit State Function

The reliability against fatigue failure of the considered rotor blade in edgewise bending is analyzed for the cyclic loading caused by wind over the design life. For this purpose, a limit state function is defined

$$g(\mathbf{X}) = 1 - D(\mathbf{X}) = 1 - \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{25}$$

in which D is the predicted cumulative fatigue damage expressed through the Miner's sum as defined in a previous section, and $\mathbf{X}$ denotes the vector of stochastic variables which include the variables $\mathbf{U}$ that represent the statistical uncertainty in the loading, and the variables ($\log_{10}K$,m,e) that represent the variability and statistical uncertainty in the resistance.

4. RELIABILITY ANALYSES

The reliability is the complement of the failure probability

$$P_F = P[g(\mathbf{X}) \le 0] \tag{26}$$

which refers to fatigue failure during a design lifetime of 20 years and may be expressed in terms of the reliability index $\beta = -\Phi^{-1}(P_F)$. The reliability is computed by means of a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The probabilistic analysis program PROBAN, see Tvedt (1989), is used for this purpose. The results of the reliability analysis are presented in Table 2.

By examination of the resulting importance factors reported in the fourth column of Table 2, it appears that the inherent variability in the fatigue life as represented by the uncertainty in the residual e of the ε–N curve is by far the single most important uncertainty source. As much as 80% of the total uncertainty importance is attributed to this resistance variable, while the other ε–N curve variables m and $\log_{10}K$ vouch for another 19% of the uncertainty importance. This leaves less than 1% uncertainty importance ascribed to the uncertainty in the load variables U_0 and U_1.

Table 2 Results of Reliability Analysis 20-Year Lifetime Fatigue in Edgewise Bending Rotor Blade, W=0.0028 m^3			
Probability of Failure P_F=1.11·10^{-4} Reliability Index β=3.69			
Variable	Distribution	Design point **x***	Importance factor α^2
U_0	Normal	−0.0404	0.008
U_1	Normal	0.0328	
$\log_{10}K$	Normal	−11.589	0.193
m	Normal	7.525	
e	Normal	−1.3082	0.799

A comparison with results from a similar study for a similar wind turbine in flapwise bending, see Ronold (1997), is appropriate here. For both edgewise bending and flapwise bending, the dominating uncertainty importance is found to be that associated with the inherent variability in the fatigue life as represented by the residual e of the ε–N curve. For both analyses, importance factors for this residual are found to be well beyond 50%. As regards the remainder of the total importance, namely that which is ascribed to variables, whose distributions represent statistical uncertainty, a couple of significant differences are found between the analyses for edgewise and flapwise bending.

First, the analysis for edgewise bending gives considerably higher importance factor for m and $\log_{10}K$ than does the analysis for flapwise bending. This is believed to be due to the fact that the edgewise bending is dominated by gravity load, which utilizes a particular part of the ε–N curve, which is not central with respect to the data that are used to estimate this curve. Flapwise bending is not dominated by such a particular load and utilizes the ε–N curve more evenly, such that the result of the analysis is much less sensitive to the uncertainty in m and $\log_{10}K$.

Second, the analysis for edgewise bending gives considerably less importance factor for the statistical uncertainty in the load distribution than does the analysis for flapwise bending. This is believed to be due to the fact that the statistical uncertainty associated with the load distribution is less in edgewise bending than in flapwise bending for those wind climate bins that contribute the most to the cumulative damage. Reference is made to the findings, discussed in a previous section, that in edgewise bending the largest statistical uncertainty is in the low-speed wind climate bins, whereas in flapwise bending the largest statistical uncertainty is in the high-speed wind climate bins.

An inspection of the computational results reveals that the major contribution to the cumulative damage is ascribed to the effect of the gravitational component of the loading. About 85% of the cumulative damage is owing to about $3\cdot10^8$ load cycles in a bending moment range interval (200 kNm; 260 kNm) near the pure, deterministic gravitational bending moment range of approximately 221 kNm. Note that this number of cycles corresponds approximately to the number of rotations.

5. CALIBRATION OF PARTIAL SAFETY FACTORS

5.1 Philosophy

It is of interest to demonstrate how reliability analysis results, obtained as outlined in the previous chapters, play a role in codified practice and design. Calibration of partial safety factors for design is an important application. With the first-order reliability method available, it is possible to determine sets of equivalent partial safety factors which result in rotor blade designs with a prescribed reliability. As a first step, a target reliability index β_T must be selected.

The choice for the target reliability index can be derived from a utility-based feasibility assessment in a decision analysis, or by requiring that the safety level as resulting from the design by a reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society. A range of target reliability indices will be considered in the following.

In the case of a prescribed reliability level, which is different from the one that results from an actually executed reliability analysis of a wind turbine rotor blade, the geometrical quantities of this blade must be adjusted. The adjustment is made in such a way that the required reliability level will result from a new reliability analysis of the modified blade. The geometrical quantities, which can be adjusted to achieve a specified reliability level, are sometimes denoted design parameters. It is most practicable to operate on just one such design parameter when adjusting the design in order to reach the specified reliability level. For a rotor blade, the most practicable parameter to adjust is the section modulus W, which is a function of the cross-sectional properties of the blade at the root.

To cover a range of target reliabilities, a series of reliability analyses is carried out for a range of values for the section modulus W. This gives the reliability index β as a function of the section modulus W, $\beta=\beta(W)$. The target reliability index corresponds to a requirement to the annual probability of failure. For a fatigue problem as the present, where failure is associated with the accumulation of a damage over time, the annual probability of failure will increase from one year to the next during the design life. The requirement to the failure probability shall be met in all years during the design life. This implies that if the requirement is fulfilled for the last year during the design life, it will be fulfilled for all years. The design life is taken as 20 years. For various section moduli W, the reliability index of interest for the code calibration is then calculated as

$$\beta = -\Phi^{-1}(P_{F,20} - P_{F,19}) \tag{27}$$

in which $P_{F,20}$ is the probability of fatigue failure in 20 years, and $P_{F,19}$ is the probability of failure in 19 years, calculated as outlined above.

5.2 Characteristic Values of Governing Variables

Characteristic values have to be selected for the governing load and resistance variables. For design in ultimate loading, the 98% quantile of the annual maximum load is traditionally used as the characteristic load value. For design in fatigue loading, this has hardly any meaning, as a characteristic load distribution for the design life needs to be selected rather than such a single characteristic load value. The governing load distribution is a compound distribution of the bending moment range over the design life of the rotor blade and has contributions from many moment range distributions conditioned on different 10-minute wind climates (U_{10},I_T). For simplicity, an idealized characteristic moment range distribution is adopted here,

$$X = k_R X_0 (1 - \frac{\log_{10} n}{\log_{10} N_r}) + X_G \tag{28}$$

in which X denotes the moment range which is exceeded in n stress cycles during the design life T_L, N_r=$f_r T_L$ is the number of rotor cycles in this life, f_r denotes the rotor frequency, X_0 is a characteristic bending moment whose derivation is described below, X_G is the bending moment range owing to gravity, and k_R is a scaling factor. A value of k_R =0.918 is applied in the following. This value has been calibrated as the one that will lead to a cumulative damage approximately equal to the median damage for the true compound moment range distribution derived from the observed conditional moment range distributions as described above. Note that a smaller value for k_R is required here for edgewise bending than for flapwise bending of the similar rotor blade as reported in Ronold (1997). This difference is much owing to the fact that the true load distributions in edgewise bending and flapwise bending are different, in particular because of a significant effect of gravity in edgewise bending. The effect of gravity in edgewise bending could, as an alternative, have been accounted for in other ways than by adjusting the value of k_R, for example according to an approach described in Dansk Ingeniørforening (1992).

In accordance with the Danish code, see Dansk Ingeniørforening (1992), the characteristic bending moment is defined as

$$X_0 = \frac{\rho}{2} w^2 c C_L \frac{R^2}{3} \tag{29}$$

in which R is the radius of the rotor measured from the center of the rotor to the tip, c is characteristic chord length at $2R/3$, C_L=1.5 is a lift coefficient at $2R/3$, ρ=1.28 kg/m^3 is the density of air, and w is a reference wind speed defined by

$$w^2 = (\frac{4\pi}{3} f_r R)^2 + v_0{}^2 \tag{30}$$

where f_r is the rotor frequency as before, and v_0 is the 10-minute mean wind speed at stalling of the entire rotor blade. For the considered 600 kW wind turbine, the following numbers apply: T_L=20 years, f_r =0.48 sec^{-1}, N_r =0.303·10^9 rotations, R=21.5 m, c=1.00 m, and v_0=14.6 m/sec. The bending moment range due to gravity is taken as X_G=220.6 kNm for the considered turbine.

For ε–N curves, it is a standard approach to select the characteristic ε–N curve as the curve that results when the estimated ε–N curve is shifted to the left by a distance equal to two times the standard deviation of the residual e, hence

$$\begin{aligned}\log_{10} N &= E[\log_{10} K] - E[m]\log_{10}\varepsilon - 2\sigma_e \\ &= -12.372 - 7.912\log_{10}\varepsilon - 2\cdot 0.396 \\ &= -13.164 - 7.912\log_{10}\varepsilon\end{aligned} \tag{31}$$

when values for the considered rotor blade laminate are substituted. Reference is made to Det Norske Veritas (1984) and Dansk Ingeniørforening (1992).

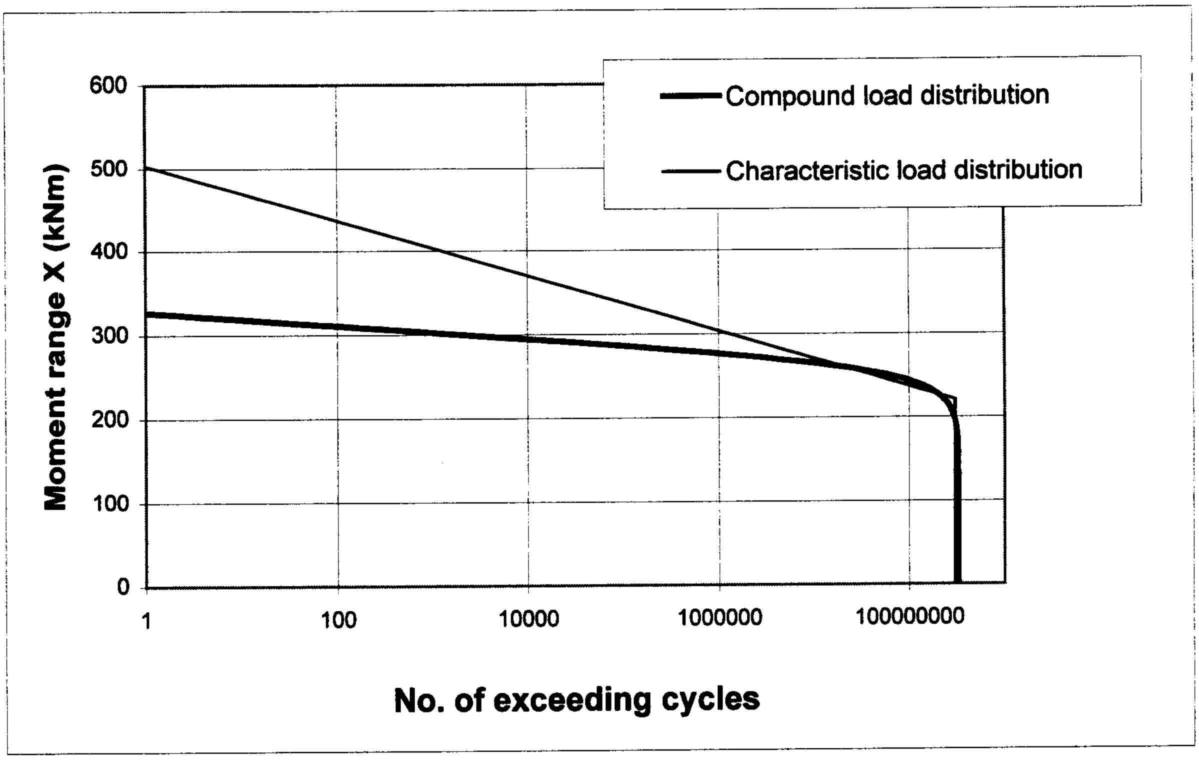

Figure 2 Comparison between data-based compound load distribution and corresponding simplified characteristic load distribution; number of cycles n with bending moment range in excess of X is shown for a 20-year design life

Figure 2 shows a comparison between the data- and model-based compound lifetime distributions of bending moment ranges, derived as described in Section 2.1, and the simplified characteristic moment range distribution, expressed as described in this section. It appears that the two distributions are fairly close together for values of the bending moment range near the value of the gravity component, i.e., in the region where the major part of the fatigue damage is produced. This is reassuring. The two distributions are seen to deviate quite a bit for larger values of the bending moment range. For a particular number of exceeding cycles, the characteristic load distribution implies a larger bending moment range than that implied by the compound distribution. This discrepancy is not as drastic as it may appear. It is the natural result of the calibration of the characteristic load distribution to the compound distribution as described above. When used for damage prediction, the discrepancy compensates for the fact that the characteristic distribution is based on a total number of load cycles which is about 8% less than that of the compound distribution. This difference in total number of load cycles between the two distributions is hardly visible in Figure 2 owing to the logarithmic scale. Further, the bending moment ranges, for which the largest discrepancies between the two distributions are found, are associated with such low numbers of cycles that their contributions to the cumulative damage are insignificant, anyway.

5.3 Partial Safety Factors and Design Load and Resistance Properties

Two partial safety factors are introduced. A load factor γ_f greater than 1.0 is applied as a factor on all load values of the characteristic moment range distribution and the relation $S=X/W$ between stress range S and moment range X is substituted such that the design stress range distribution becomes

$$S = \frac{\gamma_f}{W}(k_R X_0(1-\frac{\log_{10} n}{\log_{10} N_r}) + X_G) \tag{32}$$

It may be argued that the load factor should only be applied to the X_0-dependent term and not to the gravity term X_G, because the gravity term is usually well determined without any associated uncertainty. It is, however, practical to apply the load factor also to the gravity term as implied by Eq. (32), and this format is adopted in the following.

Correspondingly, γ_m is a material factor greater than 1.0. For any given number of cycles to failure, the characteristic strength is divided by this number to give the design strength. Recall the relationship $S=2E\varepsilon$ between stress range and strain amplitude. This implies that the design S–N curve becomes

$$\log_{10} N = -13.164 - 7.912\log_{10}(\frac{S}{2E}\gamma_m) \tag{33}$$

5.4 Calibration

For the same series of values of the section modulus W that was used for the reliability analyses, deterministic structural analyses are carried out. For each value of W, the accumulated damage D is calculated according to Eq. (14) on the basis of values of Δn predicted from the design stress range distribution in conjunction with values of N predicted from the design S–N curve. Pairs of partial safety factors (γ_f, γ_m) are determined in such a way that this accumulated damage becomes exactly equal to the limit value of 1.0 that indicates failure. This is conveniently done by calculating the accumulated damage D for many trial pairs (γ_f, γ_m) and picking those pairs for which D=1.0 results. For each value of W, there will be an infinite number of pairs (γ_f, γ_m) that will lead to D=1.0. This is a result of the form of the limit state function for fatigue failure in edgewise bending and implies an arbitrariness in selecting the partial safety factors (γ_f, γ_m). With the present code format, by which all stress ranges are multiplied by the load factor γ_f, the requirement to (γ_f, γ_m) turns out to be on their product. If the code format had been different, such that the deterministic part of the stress ranges caused by gravity were not multiplied by the load factor γ_f, then the requirement to (γ_f, γ_m) would have become more complex. Hence, with the chosen code format, the result of this exercise performed for many section modulus values W is a required partial safety factor product $\gamma_f\gamma_m$ as a function of the section modulus W, $\gamma_f\gamma_m = \gamma_f\gamma_m(W)$.

The result of the structural reliability analyses, $\beta=\beta(W)$, and the result of the deterministic structural analyses, $\gamma_f\gamma_m = \gamma_f\gamma_m(W)$, are combined by elimination of the section modulus W to give the reliability index β as a function of the calibrated partial safety factor product $\gamma_f\gamma_m$, $\beta=\beta(\gamma_f\gamma_m)$, see Figure 3, in which β refers to the reliability during the last year of a twenty-year design life.

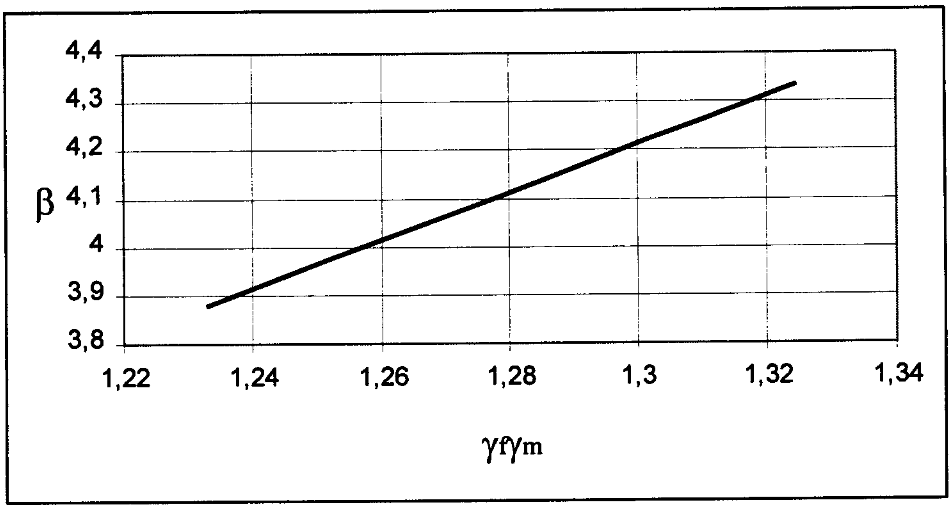

Figure 3 Reliability index β vs. calibrated partial safety factor product $\gamma_f\gamma_m$

As an example, consider a design situation where brittle failure of a non-redundant structure is at stage and has less serious consequence. This would be a reasonable classification for a rotor blade design against fatigue where human life is at negligible risk. According to Nordic Committee on Building Regulations (1978), the requirement to the annual failure probability for design under such a classification is 10^{-5}. The corresponding target reliability index is β_T=4.265. This target reliability index has to be met during each and every year during the design life of the wind turbine. This implies that if it is met during the last year of the design life, it will implicitly be met in all other years as well.

For a target reliability index β_T=4.265, Figure 3 gives a requirement to the product of the partial safety factors

$$\gamma_f\gamma_m = 1.310 \tag{34}$$

As discussed above, an infinite number of possible choices for the set of partial safety factors (γ_f, γ_m) exist for each β value, as the requirement is on their product. A robust choice of partial safety factors is usually a set which leads to design values of stresses and strengths as close as possible to the design point values resulting from the corresponding reliability analysis. This is so because the design point of the reliability analysis represents the most likely outcome of the governing stochastic variables at failure.

From the investigation of the results of the reliability analysis in a previous section, it appears that there is next to no uncertainty importance associated with the uncertainty in the applied loading. This corresponds to design point values of the uncertain load variables close to the respective mean values of these variables. This makes it natural to set the load factor equal to unity,

$$\gamma_f=1.00 \tag{35}$$

and the material factor then becomes

$$\gamma_m=1.31 \tag{36}$$

One nice feature of this particular choice of partial safety factors is that it implies that the gravity term in the expression for the characteristic stress range distribution actually achieves the safety factor of 1.0 that could have been prescribed with reference to the deterministic nature of this term. In other words, a common safety factor for load has been chosen, which honors the lack of uncertainty associated with the gravity term in the load distribution, and which thus makes it unnecessary to distinguish between different load factors for the different terms that constitute the load distribution.

6. SUMMARY AND CONCLUSIONS

The design of a wind turbine rotor blade against fatigue failure in edgewise bending has been considered. The load distribution in the design life has been modelled on the basis of observed distributions of bending moments at the blade root of an instrumented prototype rotor blade subjected to wind loads. Statistical uncertainty in the distribution parameters has been estimated and taken into account. The resistance has been modelled in terms of an ε–N curve. Uncertainties in the variables that describe this curve have been estimated and have also been taken into account. The cumulative damage that eventually leads to a fatigue failure has been predicted according to a Miner's sum formulation.

The models for load, resistance, and cumulative damage have been used as a basis for defining a limit state function for fatigue failure, and a first-order reliability analysis of the considered rotor blade against such a failure in edgewise bending has been carried out. The reliability analysis has been interpreted with respect to the probability of failure as well as identification of important uncertainty sources. The inherent variability in the fatigue life as represented by the uncertainty in the residual of the ε–N curve is found to be the single most important uncertainty source.

A reliability-based calibration of partial safety factors for design of the rotor blade against fatigue in edgewise bending has been carried out. A load factor γ_f has been applied to all stresses, and all strengths have been divided by a material factor γ_m. A target lifetime reliability corresponding to an acceptable annual probability of failure of 10^{-5} has been applied for the calibration. Based on a specific choice of characteristic values for load and resistance, a requirement to the product of load factor and material factor $\gamma_f\gamma_m$ =1.31 has come out. Based on the importance information of the underlying reliability analysis, a particular robust set of partial safety factors that fulfill this requirement has been determined, hence γ_f=1.00 and γ_m=1.31. This reflects that literally all uncertainty importance associated with the fatigue analysis in edgewise bending is found to be ascribed to the variability and uncertainty in the material properties.

It is emphasized that the reliability-based safety factor calibration presented herein is site and wind-turbine specific and only applicable to edgewise bending of rotor blades. Different safety factors may result for different sites, different wind turbines, and different blade materials. Similar calibrations can be carried out for other wind turbines at other sites. A common set of partial safety factors for a class of wind turbines, sites, and materials can then be optimized in dependence of the expected demand for each individual combination of wind turbine, site, and material within the class. Future work is suggested to be devoted to investigations of a series of wind turbines for different sites and blade materials with the ultimate goal of developing a reliability-based optimal design code for rotor blades in flapwise as well as edgewise bending.

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TECHNICAL REPORT

Client :	Danish Energy Agency through Risø National Laboratory
Title :	Calibration of Partial Safety Factors for Design of Wind-Turbine Rotor Blades against Fatigue Failure in Flapwise Bending – 450 kW Turbine
Report No. :	99-3512

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DNV

REPORT

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December 3, 1999	**OCT760**	**76010202**	**Research**

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Arne E. Løken

Client, Sponsor

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Summary

A probabilistic model for evaluation of the safety of a wind-turbine rotor blade against fatigue failure in flapwise bending is presented. The model accounts for uncertainty and variability in load and resistance. The model is applied in conjunction with a first-order reliability method to perform a structural reliability analysis of one of the rotor blades in a particular, site-specific wind turbine. The turbine selected for this purpose is a 450 kW turbine. The probability of fatigue failure in flapwise bending of one of the rotor blades of this wind turbine over a twenty-year design life is calculated. It is demonstrated how the reliability analysis results can be used to calibrate partial safety factors for load and resistance for use in conventional deterministic fatigue design.

DNV Rep.No.	Subject Group
99-3512	B3, B4, F1, K0

Title of Report

CALIBRATION OF PARTIAL SAFETY FACTORS FOR DESIGN OF WIND-TURBINE ROTOR BLADES AGAINST FATIGUE FAILURE IN FLAPWISE BENDING – 450 KW TURBINE

4 Indexing terms

- Structural Reliability
- Code Calibration
- Fatigue
- Wind Turbines

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1. INTRODUCTION

Wind-turbine rotor blades exposed to wind loading are vulnerable to cumulative fatigue damage owing to the cyclic nature of the loading. The wind speed that causes bending of the rotor blades exhibits a natural variability, such that the load amplitudes become random. The *S–N* curve that gives the number of stress cycles to failure and represents the resistance of the rotor blade material is encumbered with uncertainty owing to a limited number of test specimens as well as variability from one specimen to another.

Partial safety factors are used in structural design as factors on characteristic values of governing load and resistance quantities to account for variability and uncertainties in these quantities.

This report demonstrates how a structural reliability method can be applied as a rational means to analyze a wind-turbine rotor blade with respect to fatigue in flapwise bending, and to establish partial safety factors for design of such rotor blades against fatigue failure. A site-specific wind turbine of a prescribed make is considered, here a 450 kW turbine with 17.5 m rotor radius. Probabilistic models for the wind loading and its transfer to bending stresses are established together with a stochastic representation of the material resistance. The event of fatigue failure in flapwise bending is considered as based on a Miner's sum formulation for cumulative damage.

2. THEORY FOR LOAD, RESISTANCE, AND CUMULATIVE DAMAGE

2.1 Wind Climate and Load Distribution for Rotor Blade

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the turbulence intensity I_T. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \qquad (1)$$

in which k and A are site- and height-dependent coefficients.

The standard deviation σ_U of the wind speed depends on the 10-minute mean wind speed U_{10}. The distribution of σ_U conditioned on U_{10} can be well represented by a lognormal distribution

$$F_{\sigma_U|U_{10}}(s) = \Phi(\frac{\ln s - h_0}{h_1}) \qquad (2)$$

in which $\Phi()$ denotes the standard Gaussian cumulative distribution function, and the coefficients h_0 and h_1 depend on U_{10} as follows

$$h_0=d_1+d_2\ln(U_{10}+d_3) \tag{3}$$

$$h_1^2=d_4+d_5\exp(-d_6U_{10}) \tag{4}$$

A similar model can be found in Ronold and Larsen (1999), and an example of data and fitted model is given later in this report. The turbulence intensity I_T is defined as the standard deviation of the wind speed divided by the mean wind speed U_{10} and represents the gustiness of the wind about this mean, hence $I_T=\sigma_U/U_{10}$. The (U_{10},I_T) space is referred to in the following. This space is discretized into a number of bins of approximately constant values of U_{10} and I_T.

One rotor blade is considered in the following. Let X denote the bending moment range at the blade root in flapwise bending. One bending moment range is associated with each load cycle, and load cycles are identified by rain-flow counting. By means of the rain-flow counting technique, observed bending moment histories, each of 10-minute duration, are converted to 10-minute records of the bending moment range X on histogram form. Within each recorded 10-minute interval, also the wind climate parameters U_{10} and I_T are recorded. The (U_{10},I_T) space is discretized in a number of bins as described before. All 10-minute records of X are sorted according to U_{10} and I_T, i.e., each record is localized to a particular bin (U_{10},I_T) in the adopted discretization of the (U_{10},I_T) space. For a particular bin (U_{10},I_T) there will be a total of M 10-minute records of X, and M varies from bin to bin. These M records are used to give an estimate of the short-term distribution of X conditioned on (U_{10},I_T), i.e., $X|(U_{10},I_T)$, on discretized form. The number of load cycles n_{10} in each 10-minute interval is also observed and depends on U_{10} and I_T.

Because the distribution of $X|(U_{10},I_T)$ is encumbered with uncertainty owing to limited data for its estimation, it is desirable to parametrize the distribution and represent this uncertainty in reliability analyses as uncertainty in the distribution parameters. Such a parametrization is useful also when it is desirable to extrapolate the distribution beyond the range of the available data. The distribution of $X|(U_{10},I_T)$ can be parametrized in terms of its statistical moments. The first three statistical moments are used for this purpose. These moments are the mean value a_1, the standard deviation a_2, and the skewness a_3. Their expected values $E[a_i]$, i=1,2,3, can be estimated based on the observed discretized version of the conditional distribution of $X|(U_{10},I_T)$. Their standard deviations $D[a_i]$, i=1,2,3, and also their correlation matrix ρ can be estimated by a resampling technique such as the jackknife or the bootstrap, see Efron and Tibshirani (1993).

It is assumed that the expected values and standard deviations of a_1, a_2, and a_3 conditioned on U_{10} and I_T are adequately represented by the polynomial surfaces

$$E[a_i]=b_{0i}+b_{1i}U_{10}+b_{2i}U_{10}^2+b_{3i}I_T+b_{4i}I_T^2 \tag{5}$$

$$D[a_i]=c_{0i}+c_{1i}U_{10}+c_{2i}U_{10}^2+c_{3i}I_T+c_{4i}I_T^2 \tag{6}$$

in which the coefficients b_{ji} and c_{ji}, j=0,...4, i=1,2,3, are determined by least-squares regressions of all estimated mean values and standard deviations of a_1, a_2, and a_3 over the (U_{10},I_T) space.

$$a_i = E[a_i] + U_i D[a_i], \quad i = 1,2,3 \tag{7}$$

in which $\mathbf{U}=(U_1, U_2, U_3)^T$ is a three-dimensional normally distributed variable with zero mean, unit variance, and correlation matrix ρ. Note in this context that U_i is standard notation for a standard normally distributed variable within the field of structural reliability and is not to be confused with any wind speed. Note also that the vector $\mathbf{U}=(U_1, U_2, U_3)^T$ represents the statistical uncertainty in the three moments a_1, a_2, and a_3 owing to the limited data available for their estimation.

Above, the statistical moments a_1, a_2, and a_3 of the available measured data for the bending moment range X at the blade root in flapwise bending have been dealt with. However, no statement has so far been made with respect to the distribution of the bending moment amplitudes themselves, neither in the short term, conditional on a particular wind climate (U_{10},I_T), nor in the long term such as over the design life of the rotor blade. A model for the distribution of the bending moment range X is therefore dealt with in the following.

Load response amplitudes are often seen to have marginal distributions, which are close to Weibull distributions. The bending moment range is two times such a load response amplitude. Based on the first three moments a_1, a_2, and a_3 of the distribution of the conditional bending moment range $X|(U_{10},I_T)$, this distribution can be modelled as a quadratic expansion of a parent Weibull-distributed variable U_W. The parent Weibull-distributed variable U_W is chosen such that it has the same mean a_1 and the same standard deviation a_2 as the distribution of $X|(U_{10},I_T)$ which is to be modelled. For the case that the skewness of U_W is smaller than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a softening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa(U_W + \xi U_W^{\,2}) \tag{8}$$

For the case that the skewness of U_W is greater than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a hardening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa \frac{\sqrt{1+4\xi U_W} - 1}{2\xi} \tag{9}$$

In both cases, the model is referred to as a quadratic Weibull model, and the coefficients ξ, κ, and $x_{\min}$ are chosen such that the mean value a_1, the standard deviation a_2, and the skewness a_3 of the distribution of $X|(U_{10},I_T)$ are all preserved. The quadratic Weibull model may be thought of as a generalized or distorted Weibull distribution. Reference is made to Lange and Winterstein (1996). Note that the quadratic Weibull distribution provides a better fit to data, and in particular a better representation of the important upper tail of the distribution, than the distorted lognormal distribution used by Ronold et al. (1994) in a first approach to a parametrized representation of the load range distribution. The distorted lognormal distribution, obtained by a logarithmic Hermite polynomial expansion of a parent Gaussian distribution, is known to have a too heavy upper tail which, as commented by Ronold et al. (1994), leads to overprediction of upper quantiles of the bending moment ranges and thereby of the high-range stresses.

Note also that the quadratic Weibull model provides results very close to those obtained by a cubic Weibull model which preserves the first four statistical moments of the distribution of $X|(U_{10},I_T)$, including the kurtosis a_4, but which is computationally much more cumbersome and time-consuming and therefore less attractive, see Ronold et al. (1996). The accuracy of predictions made by means of the quadratic Weibull models of Eqs. (5) and (6) is further dealt with later.

The section modulus at the rotor blade at the blade root is W, and the stress range S corresponding to the moment range X is $S=X/W$. When a suitable discretization of the stress range space is introduced, then either Eq. (5) or Eq. (6), depending on the value of the skewness a_3, can be applied in conjunction with the distribution function of the parent Weibull variable U_W to calculate the probability content of each interval ΔS of this discretization. The corresponding number of cycles within each such interval in a 10-minute period can be determined as this probability content times the total number of cycles $n_{10}(U_{10},I_T)$.

Integrating contributions from all possible 10-minute wind climate bins (U_{10},I_T), weighted according to the quoted Weibull distribution for U_{10} and the lognormal distribution for I_T, this can be used to establish a distribution of the stress range S over the design life T_L of the rotor blade. Here T_L is taken as 20 years. This compound lifetime distribution of the stress range S can be expressed in terms of the number of stress cycles n whose associated stress range exceeds a level S during the design life of the rotor blade. An example of such a lifetime distribution is given later. The compound lifetime distribution of stress ranges can, in turn, be used to calculate the number of stress cycles Δn within an interval ΔS of the discretized stress range space.

The number of cycles $n_{10}(U_{10},I_T)$ in a 10-minute interval is also encumbered with uncertainty. However, the coefficient of variation is inversely proportional with the square-root of the interval length, such that when $n_{10}(U_{10},I_T)$ is scaled to give the number of cycles over a long time span such as a 20-year design life, the uncertainty in this number becomes insignificant and can be ignored. In the reliability analysis $n_{10}(U_{10},I_T)$ is therefore left as a deterministic quantity only dependent on U_{10} and I_T.

2.2 Fatigue Strength and *S–N* curve

For a given stress range S, the number of cycles N to failure is generally expressed through an S–N curve, $N=BS^{-k}$. However, in tests of composite materials for use in rotor blades, the strain amplitude ε is usually measured rather than the stress range S. Hence, for such materials the number of cycles N to failure is expressed through an ε–N curve. This curve can be expressed by the following relationship

$$\log_{10} N = \log_{10} K - m\log_{10} \varepsilon \tag{10}$$

in which K and m are coefficients. This gives a linear model for $\log_{10}N$

$$\log_{10} N_i = \log_{10} K - m\log_{10} \varepsilon_i + e_i \ , \ i=1,...n \tag{11}$$

in which the pair ($\log_{10}K$, m) describes the expected behavior and can be estimated by a linear regression analysis based on n observed data pairs (ε_i,N_i). The zero-mean terms e_i denote residuals that represent local variations from test specimen to test specimen, or from one point of the rotor blade to another. The standard deviation σ_e of the residuals e_i will result as a byproduct of the regression analysis, and so will the standard deviations and correlation coefficient of $\log_{10}K$ and m. The stress range S that corresponds to the strain amplitude ε can be expressed as $S=2E\varepsilon$, where E denotes the modulus of elasticity of the material in the direction of the loading. The modulus of elasticity is idealized as a constant here, but may in general vary with the magnitude of the strain. A refined representation with such a variation included would be desirable. Further, the possible effect of a non-zero mean stress has been ignored, mainly because of limitations in available test data. Scale effects from test specimen to prototype and long-term environmental degradation effects owing to exposure to moisture and ultraviolet light have been left out of consideration. Also the inclusion of such effects would be desirable if data would permit.

2.3 Cumulative Damage and Failure Criterion in Fatigue

According to Miner's rule, fatigue failure in a structural material is defined to occur when the accumulated damage D exceeds 1.0, where D is defined as

$$D=\sum_{i=1}^{k}\frac{\Delta n(S_i)}{N(S_i)} \tag{12}$$

Here, Δn is the number of load cycles at stress range S in the lifetime of the rotor blade, and N is the number of cycles to failure at this stress range. The sum is over all stress ranges S_i in a sufficiently fine discretization of the stress range space.

3. PROBABILISTIC AND DETERMINISTIC MODELLING

The reliability of a site-specific wind turbine against fatigue failure of one of its rotor blades in flapwise bending is considered. The reliability is computed by a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The input to the reliability analysis consists of a limit state function, specified in terms of a set of basic variables, which consist of stochastic variables as well as deterministic parameters. Furthermore, the statistical distributions of the stochastic variables must be given, and the values of the deterministic parameters must be specified. The following sections describe the stochastic variables, the deterministic parameters, and the limit state function. Separate sections are devoted to make and site of wind turbine, environmental loading, fatigue strength, model uncertainty, and'limit state function.

3.1 Wind Turbine Characteristics

A 450 kW wind turbine with rotor radius R=17.5 m and hub height z=35 m is considered. The section modulus of the rotor blade at the blade root in flapwise bending is W=0.0005 m^3.

3.2 Environmental Loading

The wind turbine is considered for a location whose wind loading regime is characterized by a scale parameter A=9.1 m/sec and a shape parameter k=1.9 in the long-term Weibull distribution of the 10-minute mean wind speed U_{10}.

As described in a previous section, the distribution of the standard deviation σ_U of the wind speed, conditioned on the 10-minute mean wind speed U_{10}, can be represented by a lognormal distribution

$$F_{\sigma_U|U_{10}}(s) = \Phi(\frac{\ln s - h_0}{h_1}) \quad (13)$$

Based on the available wind climate data from the considered location, the coefficients h_0 and h_1 are represented as functions of U_{10} as follows

$$h_0 = -3.1530 + 1.3793 \cdot \ln(U_{10} + 1.7434)$$

$$h_1^2 = 0.005173 + 0.4047 \cdot \exp(-0.1980 U_{10}) \quad (14)$$

when U_{10} is given in units of m/sec. These relationships and the data that they are based on are represented in Figure 1.

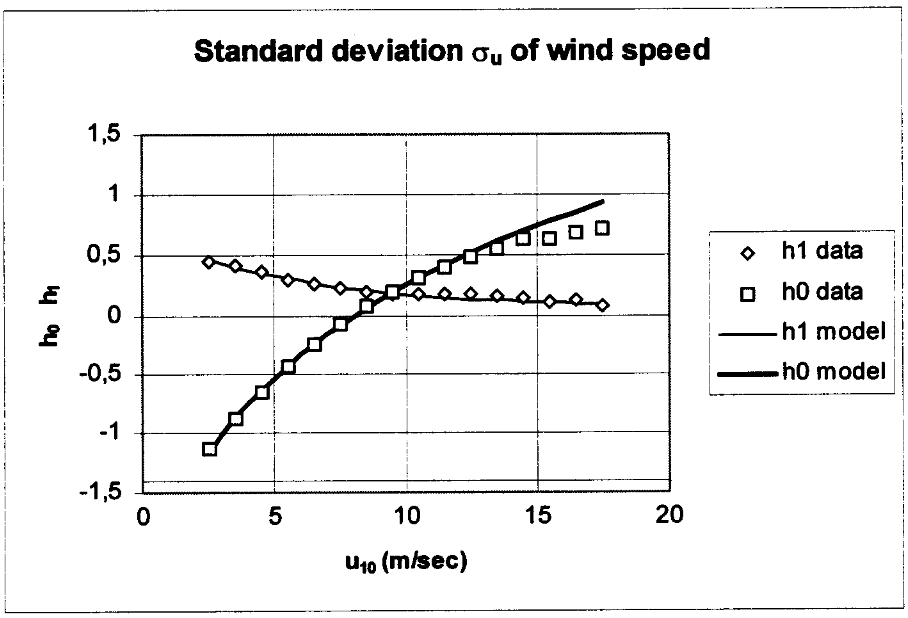

Figure 1 Coefficients h_0 and h_1 in distribution model for standard deviation σ_U

A total of 891 10-minute records of flapwise bending moment ranges X for various wind climate realizations (U_{10},I_T), considered fixed within each 10-minute interval, are available from measurements on a prototype turbine. The (U_{10},I_T) space is discretized into bins of widths ΔU_{10}=2 m/sec and ΔI_T=0.01, and each of the 891 10-minute records of X is allocated to the appropriate bin.

For each bin (U_{10},I_T), the available M 10-minute moment range records are merged, and the observed moment ranges X are sorted in increasing order. From this, the cumulative distribution function of $X|(U_{10},I_T)$ is derived, and its first three moments, here denoted a_1 through a_3, are estimated. The standard deviations of these three moments are obtained by jackknifing the M 10-minute records. As stated in a previous section, the following model is chosen to represent the coefficients a_1 through a_3

$$a_i = E[a_i] + U_i D[a_i], \quad i = 1,2,3 \tag{15}$$

in which the mean value $E[a_i]$ and standard deviation $D[a_i]$ of the ith moment a_i are represented as

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^2 + b_{3i}I_T + b_{4i}I_T^2 \tag{16}$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^2 + c_{3i}I_T + c_{4i}I_T^2 \tag{17}$$

Table 1 Estimated Coefficients in Polynomial Model for $E[a_i]$ and $D[a_i]$
Measured bending moment ranges

i	b_0	b_1	b_2	b_3	b_4	c_0	c_1	c_2	c_3	c_4
1	2.190	0.363	0.0199	-2.813	54.35	0.743	-0.063	0.0036	-9.012	52.538
2	5.273	-0.169	0.0473	-4.988	47.38	0.973	-0.083	0.0044	-11.04	62.162
3	2.786	-0.268	0.0097	-10.66	49.13	0.284	-0.027	0.0011	-2.721	17.686

Units of coefficients are consistent with bending moment ranges quoted in units of kNm and wind speeds in m/sec

The polynomial coefficients in these expressions are determined by a least-squares regression of the available data and are presented in Table 1. The stochastic variables denoted $\mathbf{U}=(U_1,U_2,U_3)^T$ represent the statistical uncertainties in the bending moment range distributions and follow a three-dimensional normal distribution with mean values 0.0, standard deviations 1.0, and a correlation matrix which is estimated to be

$$\rho = \begin{bmatrix} 1.000 & 0.901 & -0.310 \\ 0.901 & 1.000 & -0.037 \\ -0.310 & -0.037 & 1.000 \end{bmatrix} \tag{18}$$

For measured bending moment ranges, the number of aerodynamic stress cycles n_{10} in a 10-minute interval is represented as a function of (U_{10},I_T) as follows

$$n_{10}=2006.7-89.517U_{10}+4.9462U_{10}^2+4624.9I_T-30089I_T^2 \tag{19}$$

in which the coefficients are estimated by a least-squares regression from a total of 891 records of n_{10}, when U_{10} is quoted in units of m/sec.

As described in a previous section, the conditional distribution of $X|(U_{10},I_T)$, expressed in terms of a parent Weibull distribution, is used in conjunction with the long-term distributions of U_{10} and I_T as well as the number of cycles $n_{10}(U_{10},I_T)$ in 10-minute intervals to establish a compound lifetime distribution of the bending moment range X for the considered rotor blade in flapwise bending. The corresponding distribution of the bending stress range S of interest for prediction of cumulative damage by Miner's sum is easily derived by division by the resistance moment W, hence $S=X/W$.

3.3 Resistance and Stiffness of Composite Laminate

As stated in a previous section, the ε–N curve that gives the number of stress cycles N to failure as a function of the strain amplitude ε is given by the linear relationship

$$\log_{10} N = \log_{10} K - m\log_{10} \varepsilon + e \tag{20}$$

for the rotor blade laminate.

A total of 81 observed pairs (ε,N) are available from laboratory tests on specimens of a polyester laminate reinforced by five layers of combined woven glass roving and chopped strand mat with fibres oriented at 0/90 during testing and with some fibres in the load direction. A regression analysis of these data pairs according to the linear model leads to the following estimates of mean values, standard deviations, and correlation coefficient for the coefficients $\log_{10} K$ and m

$$E\begin{bmatrix}\log_{10} K\\ m\end{bmatrix}=\begin{bmatrix}-12.372\\ 7.912\end{bmatrix}\;;\qquad D\begin{bmatrix}\log_{10} K\\ m\end{bmatrix}=\begin{bmatrix}0.513\\ 0.247\end{bmatrix}\;;\qquad \rho=-0.996 \tag{21}$$

when strain amplitudes are quoted as a dimensionless absolute quantity. Under the central limit theorem the distribution of ($\log_{10}K$,m) is a bivariate normal distribution. The standard deviation of the residual term e is estimated to be σ_e=0.396. The zero-mean residual term e is represented by a normal distribution with this standard deviation. For details about the data and the tests of the laminates, reference is made to Echtermeyer et al. (1993) and Echtermeyer (1994).

A constant modulus of elasticity is used, E=29.7·10^6 kPa.

3.4 Model Uncertainty

Model uncertainty can be associated with all simplifications and idealizations made in the formulation of the engineering models that are used for analysis of fatigue damage and failure of a rotor blade in bending. One of these model uncertainties is considered here, namely that which is associated with the use of the quadratic Weibull model for representation of the distribution of the bending moment range conditional on the wind climate (U_{10},I_T). Damage predictions by the Miner sum, based on such quadratic Weibull distributions for the loading, are therefore multiplied by a random factor F_M. This random factor represents the bias and uncertainty in these damage predictions as associated with the use of the quadratic Weibull model for the conditional load distributions. For a series of 38 wind climate bins (U_{10},I_T), observations of conditional load distributions are available from measurements, and the corresponding quadratic Weibull models for these distributions have been fitted. For each bin, two damage predictions by the Miner sum have been made, the first based on the observed empirical load distribution, the second based on the fitted quadratic Weibull model, and the ratio between the two predictions has been calculated. A statistical analysis of the 38 damage ratios gives the following estimates of the mean value and standard deviation of the random model uncertainty factor F_M

$$E[F_M]=0.989 \qquad D[F_M]=0.258 \tag{22}$$

The distribution of F_M is taken as a normal distribution.

It appears that there is hardly any bias in the damage predictions by the quadratic Weibull model for the loading, as the mean value of F_M is found to be very close to 1.0. This serves to support use of the quadratic Weibull model as a rather accurate model for representation of flapwise bending moment ranges of rotor blades, in particular when considering it is based on a fit to the first three statistical moments only.

3.5 Limit State Function

The reliability against fatigue failure of the considered rotor blade in flapwise bending is analyzed for the cyclic loading caused by wind over the design life. For this purpose, a limit state function is defined

$$g(\mathbf{X}) = 1 - F_M D(\mathbf{X}) = 1 - F_M \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \tag{23}$$

in which D is the predicted cumulative fatigue damage expressed through the Miner's sum as defined in a previous section, and $\mathbf{X}$ denotes the vector of stochastic variables which include the variables $\mathbf{U}$ that represent the uncertainty in the loading, the variables ($\log_{10}K$,m,e) that represent the uncertainty in the resistance, and the variable F_M that represents the bias and uncertainty in the cumulative damage predictions as resulting from use of the quadratic Weibull model for the loading.

4. RELIABILITY ANALYSES

The reliability is the complement of the failure probability

$$P_F = P[g(\mathbf{X}) \le 0] \tag{24}$$

which refers to fatigue failure during a design life of 20 years. and may be expressed in terms of the reliability index $\beta=-\Phi^{-1}(P_F)$. The reliability is computed by means of a first-order reliability method as described in Madsen et al. (1986) and Ronold et al. (1994). The probabilistic analysis program PROBAN, see Tvedt (1989), is used for this purpose. The results of the reliability analysis are presented in Table 2.

By examination of the resulting importance factors reported in the fourth column of Table 2, it appears that the inherent variability in the fatigue life as represented by the uncertainty in the residual e of the ε–N curve is by far the single most important uncertainty source. As much as 80% of the total uncertainty importance is attributed to this resistance variable, while the other ε–N curve variables m and $\log_{10}K$ vouch for another 6% of the uncertainty importance. This leaves 10% uncertainty importance ascribed to the uncertainty in the load variables U_1, U_2, and U_3, and about 5% ascribed to the load model uncertainty factor F_M.

Table 2 Results of Reliability Analysis
20-Year Lifetime Fatigue in Flapwise Bending
Rotor Blade, W=0.0005 m^3

Probability of Failure $P_F=0.80\cdot10^{-4}$
Reliability Index β=3.775

Variable	Distribution	Design point x*	Importance factor α^2
U_1	Normal	1.0301	
U_2	Normal	1.1960	0.101
U_3	Normal	0.0257	
$\log_{10}K$	Normal	-11.978	0.057
m	Normal	7.713	
e	Normal	−1.3353	0.797
F_M	Normal	1.1954	0.045

A comparison with results from a similar study for a fairly similar wind turbine has been made, see Ronold et al. (1999). The material properties for the two wind turbines are identical, whereas the load properties are turbine-specific, relying on measurements on the individual turbines. For the similar turbine studied by Ronold et al. (1999), the uncertainty importance associated with the load variables was found to amount to only 2%, such that as much as 93% of the total uncertainty importance was found to be ascribed to the uncertainties in the material properties. This slight shift in uncertainty importance between the load variables and the resistance variables is not particularly significant, but may partly be explained by the fact that for the similar turbine about 30% more load measurements were available, thus leaving that turbine with somewhat less statistical uncertainty in the load distributions than there is in the load distributions for the present Bonus turbine.

An inspection of the computational results reveals that the major contribution to the cumulative damage is ascribed to the about 10^7 medium-amplitude stress cycles in a couple or more cycle-number decades centered about $\log_{10}N$=6 in the bending moment range distribution. This is a fairly small fraction of the total of about 10^9 stress cycles that occur over the design life of the rotor blade. The fact that such a low fraction of the applied stress cycles vouches for most of the cumulative damage can be ascribed to the value of the slope parameter m of the ε–N curve which is approximately equal to 8 for the composite laminate in the present case. Stress ranges are raised to the mth power for prediction of the number of cycles to failure. This implies that the higher the value of m, the more dominant are the high stress ranges. A comparison can be made with welded steel details whose m values are usually in the range 3-4 and whose major contribution to accumulated fatigue damage is ascribed to the low-amplitude stress cycles that correspond to the lower right part of the stress range distribution in Figure 1. These stress cycles form the majority of the total number of stress cycles over the design life. An interesting consequence of this dependency of the cumulative damage on the value of m is that if epoxy materials are considered for the rotor blade, for which m-values of up to 12 or 13 are seen, then the fatigue problem may be turned into an extreme value problem as far as the loading goes. Even for the present m-value of 8, this indicates how important a proper estimation of the upper tail of the load distribution is.

5. CALIBRATION OF PARTIAL SAFETY FACTORS

5.1 Philosophy

It is of interest to demonstrate how reliability analysis results, obtained as outlined in the previous chapters, play a role in codified practice and design. Calibration of partial safety factors for design is an important application. With the first-order reliability method available, it is possible to determine sets of equivalent partial safety factors which result in rotor blade designs with a prescribed reliability. As a first step, a target reliability index β_T must be selected.

The choice for the target reliability index can be derived from a risk acceptance criterion, or by requiring that the safety level as resulting from a design by a structural reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society. Here, as an example, consider a design situation where brittle failure of a non-redundant structure is at stage and has less serious consequence. This would be a reasonable classification for a rotor blade design against fatigue where human life is at negligible risk. According to Nordic Committee on Building Regulations (1978), the requirement to the annual failure probability for design under such a classification is 10^{-5}. The corresponding target reliability index is β_T=4.265, and the most critical year with respect to fatigue during the 20-year design life is the last year.

For a fatigue problem as the present, where failure is associated with the accumulation of a damage over time, the annual probability of failure will increase from one year to the next during the design life. The requirement to the failure probability shall be met in all years during the design life. This implies that if the requirement is fulfilled for the last year during the design life, it will be fulfilled for all years. The design life is taken as 20 years. The reliability index of interest for the code calibration is then calculated as

$$\beta = -\Phi^{-1}(P_{F,20} - P_{F,19}) \tag{25}$$

in which $P_{F,20}$ is the probability of fatigue failure in 20 years, and $P_{F,19}$ is the probability of failure in 19 years. $P_{F,20}$ is obtained exactly as outlined in the analysis above, while $P_{F,19}$ is obtained from a fully analogous analysis in which the reference period T_L is taken as 19 years instead of 20 years with all other variables kept unchanged.

In the case of a prescribed reliability level β_T, which is different from the one that results from the actually executed reliability analyses, cfr. Eq. (25), the geometrical quantities of the blade have to be adjusted such that this required reliability level results from a set of reliability analyses of the modified blade. The geometrical quantities, which can be adjusted to achieve a specified reliability level, are sometimes denoted design parameters. It is most practicable to operate on just one such design parameter when adjusting the design in order to reach the specified reliability level. For a rotor blade, the most practicable parameter to adjust is the section modulus W, which is a function of the cross-sectional properties of the blade at the root.

Note that, as a byproduct of the reliability analyses, the values of the design point e_d* of the ε $-N$ curve residual e prove useful in the following and are retained.

5.2 Characteristic Values of Governing Variables

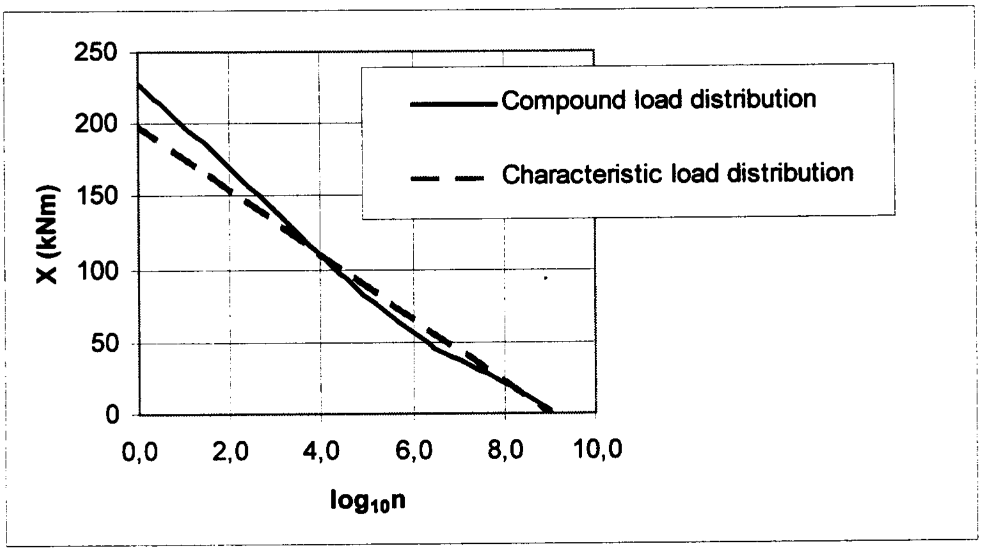

Figure 2 Comparison between data-based compound load distribution and corresponding simplified characteristic load distribution; number of cycles n with bending moment range in excess of X is shown for a 20-year design life

Characteristic values have to be selected for the governing load and resistance variables. For design in ultimate loading, the 98% quantile of the annual maximum load is traditionally used as the characteristic load value. For design in fatigue loading, this has hardly any meaning, as a characteristic load distribution for the design life needs to be selected rather than such a single characteristic load value. The governing load distribution is a compound distribution of the bending moment range over the design life of the rotor blade and has contributions from many moment range distributions conditioned on different 10-minute wind climates (U_{10},I_T). For simplicity, an idealized characteristic moment range distribution is adopted here,

$$X = k_R X_0 \left(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}\right) \tag{26}$$

in which X denotes the moment range which is exceeded in n stress cycles during the design life T_L, $N_r = f_r T_L$ is the number of rotor cycles in this life, f_r denotes the rotor frequency, X_0 is a characteristic bending moment whose derivation is described below, and k_R is a scaling factor. A value of $k_R=1.16$ is applied in the following. This value has been calibrated as the one that will lead to a cumulative damage approximately equal to the median damage for the true compound moment range distribution derived from the observed conditional moment range distributions as described above. Figure 2 shows a comparison between the data- and model-based compound lifetime distribution of bending moment ranges and the simplified characteristic moment range distribution.

In accordance with the Danish code, see Dansk Ingeniørforening (1992), the characteristic bending moment is defined as

$$X_0 = \frac{\rho}{2} w^2 c C_L \frac{R^2}{3} \tag{27}$$

in which R is the length or radius of the rotor blade measured from the center of the rotor to the tip, c is characteristic chord length at $2R/3$, $C_L=1.5$ is a lift coefficient at $2R/3$, $\rho=1.28$ kg/m^3 is the density of air, and w is a reference wind speed defined by

$$w^2 = \left(\frac{4\pi}{3} f_r R\right)^2 + v_0^2 \tag{28}$$

where f_r is the rotor frequency as before, and v_0 is the 10-minute mean wind speed at stalling of the entire rotor blade. For the considered 600 kW wind turbine, the following numbers apply: T_L=20 years, f_r=0.5833 sec^{-1}, $N_r=0.368\cdot10^9$ rotations, R=17.5 m, c=0.86 m, and v_0=14.6 m/sec.

For ε–N curves, it is a standard approach to select the characteristic ε–N curve as the curve that results when the estimated ε–N curve is shifted to the left by a distance equal to two times the standard deviation of the residual e, hence

$$\begin{aligned}\log_{10} N &= E[\log_{10} K] - E[m]\log_{10}\varepsilon - 2\sigma_e \\ &= -12.372 - 7.912\log_{10}\varepsilon - 2\cdot 0.396 \\ &= -13.164 - 7.912\log_{10}\varepsilon\end{aligned} \tag{29}$$

when values for the considered rotor blade laminate are substituted. Reference is made to Det Norske Veritas (1984) and Dansk Ingeniørforening (1992).

5.3 Partial Safety Factors and Design Load and Resistance Properties

Two partial safety factors are introduced. A load factor γ_f greater than 1.0 is applied as a factor on all load values of the characteristic moment range distribution and the relation $S=X/W$ between stress range S and moment range X is substituted such that the design stress range distribution becomes

$$S = \gamma_f k_R \frac{X_0}{W}(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}) \qquad (30)$$

Correspondingly, γ_m is a material factor greater than 1.0. For any given number of cycles to failure, the characteristic strength is divided by this factor to give the design strength. Recall the relationship $S=2E\varepsilon$ between stress range and strain amplitude. This implies that the design S–N curve becomes

$$\log_{10} N = -13.164 - 7.912\log_{10}(\frac{S}{2E}\gamma_m) \qquad (31)$$

5.4 Calibration

Based on reliability analyses for a number of trial values of the section modulus W in conjunction with references periods T_L of 19 and 20 years, it is found that a section modulus W=0.0005056 m^3 leads to a design with a reliability index β that exactly meets the requirement β_T=4.265. For this value of W, the accumulated damage D is calculated according to Eq. (12) on the basis of values of Δn predicted from the design stress range distribution in conjunction with values of N predicted from the design S–N curve. Pairs of partial safety factors (γ_f, γ_m) are determined in such a way that this accumulated damage becomes exactly equal to the limit value of 1.0 that indicates failure. This is conveniently done by calculating the accumulated damage D for many trial pairs (γ_f, γ_m) and picking those pairs for which D=1.0 results. There will be an infinite number of pairs (γ_f, γ_m) that will lead to D=1.0. This is a result of the form of the limit state function for fatigue failure in flapwise bending and implies an arbitrariness in selecting the partial safety factors (γ_f, γ_m), as the requirement turns out to be on their product. In the present case, the following requirement to the partial safety factors comes out,

$$\gamma_f\gamma_m = 1.311 \qquad (32)$$

A robust choice of partial safety factors is usually a set which leads to design values of stresses and strengths as close as possible to the design point values resulting from the corresponding reliability analyses. This is so because the design points of the reliability analyses represent the most likely outcomes of the governing stochastic variables at failure. In the following, it is outlined how such a particular robust set of partial safety factors can be chosen for the present design problem and thereby remedy the arbitrariness in the result of the calibration.

Based on the characteristic ε–N curve established in Section 5.2 above, the design ε–N curve can be expressed as

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10}(\varepsilon\gamma_m) - 2\sigma_e \tag{33}$$

Based on the results of the reliability analysis corresponding to a 20-year reference period, the ε $-N$ curve in the design point can be expressed as

$$\log_{10} N = \log_{10} K^* - m^* \log_{10}\varepsilon + e_d^* \tag{34}$$

in which $\log_{10}K^*$, m^*, and e_d^* denote design point values of the intercept $\log_{10}K$, the slope m, and the residual e, respectively, in this reliability analysis. The sought-after particular choice for γ_m is achieved by requiring this design point curve to be equal to the design curve and eliminating N. Unfortunately, this will give an expression for γ_m which is not invariant with the strain ε. As an approximation, the design point values for m and $\log_{10}K$ can be replaced by the mean values of these variables. This approximation can be justified by the relatively small uncertainty importance ascribed to these variables as determined by the reliability analysis. Hence, Eq. (34) changes to

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10}\varepsilon + e_d^* \tag{35}$$

and the sought-after particular choice for γ_m becomes

$$\gamma_m = 10^{-\frac{e_d^* + 2\sigma_e}{E[m]}} = 10^{-\frac{e_d^* + 0.792}{7.912}} \tag{36}$$

which is a requirement expressed explicitly in terms of the results of the reliability analysis corresponding to a reference period of 20 years. For W=0.0005056 m^3, which leads to a reliability index β=4.265 in the last year during the design life of 20 years, the underlying reliability analysis for fatigue failure any time during the 20-year design life gives a reliability index β=3.859 and a design point value e_d^*=−1.3645 for the ε–N curve residual e. Eq. (36) then yields the following value for the material factor

$$\gamma_m = 10^{-\frac{e_d^* + 0.792}{7.912}} = 10^{-\frac{-1.3645 + 0.792}{7.912}} = 1.18 \tag{37}$$

and the requirement to the load factor is then implied as

$$\gamma_f = \frac{\gamma_f\gamma_m}{\gamma_m} = 1.11 \tag{38}$$

The robustness in the particular set of partial safety factors that result from use of Eq. (36) is embedded in the fact that it, by its derivation in compliance with the results of the underlying reliability analysis, reflects appropriately the uncertainty importance information which is a by-product of this analysis. Reference is made to Det Norske Veritas (1992).

Note, in particular, that use of results from the reliability analysis for a 20-year reference period for the interpretation of γ_m in Eq. (36) is consistent with using a 20-year design life for calculation of design loads and for calibration of the requirement to $\gamma_f\gamma_m$.

The required load factor is found to be fairly close to 1.1, which reflects that some uncertainty importance is attributed to the uncertainty in the loading and the load model as assessed by the reliability analysis. The required material factor is correspondingly found to be about 1.2. This may seem not to be a particularly strict requirement. However, the partial safety factors are much dependent on the choices made for the corresponding characteristic values for load and resistance. In the present case, a lower-tail quantile of the resistance properties is chosen as the characteristic resistance. This implies that the characteristic resistance automatically accounts for part of the uncertainty in the resistance, and the material factor is then only meant to account for the remainder of this uncertainty, thus leaving the safety factor requirement in the present case to 1.2.

6. SUMMARY AND CONCLUSIONS

The design of a wind turbine rotor blade against fatigue failure in flapwise bending has been considered. The load distribution in the design life has been modelled on the basis of observed distributions of bending moments at the blade root of an instrumented prototype rotor blade subjected to wind loads. Statistical uncertainty in the distribution parameters has been estimated and taken into account. The resistance has been modelled in terms of an ε–N curve. Uncertainties in the variables that describe this curve have been estimated and have also been taken into account. The cumulative damage that eventually leads to a fatigue failure has been predicted according to a Miner's sum formulation.

The models for load, resistance, and cumulative damage have been used as a basis for defining a limit state function for fatigue failure, and a first-order reliability analysis of the considered rotor blade against such a failure in flapwise bending has been carried out. The reliability analysis has been interpreted with respect to the probability of failure as well as identification of important uncertainty sources. The inherent variability in the fatigue life as represented by the uncertainty in the residual of the ε–N curve is found to be the single most important uncertainty source.

A reliability-based calibration of partial safety factors for design of the rotor blade against fatigue in flapwise bending has been carried out, based on the observed load distributions. A load factor γ_f has been applied to all stresses, and all strengths have been divided by a material factor γ_m. A target reliability corresponding to an acceptable annual probability of failure of 10^{-5} has been applied for the calibration. Based on a specific choice of characteristic values for load and resistance, a requirement to the product of load factor and material factor $\gamma_f\gamma_m = 1.31$ has come out. Based on the importance information of the underlying reliability analysis, a particular robust set of partial safety factors that fulfill this requirement has been determined, hence γ_f=1.11 and γ_m=1.18.

It is emphasized that the reliability-based safety factor calibration presented herein is site and wind-turbine specific and only applicable to flapwise bending of rotor blades. Different safety factors may result for different sites, different wind turbines, and different blade materials. Similar calibrations can be carried out for such different wind turbines at various sites. A common set of partial safety factors for a class of wind turbines, sites, and materials can then be optimized in dependence of the expected demand for each individual combination of wind turbine, site, and material within the class. Future work is suggested devoted to investigations of a series of wind turbines for different sites and blade materials with the ultimate goal of developing a reliability-based optimal design code. Such a code is not to be limited to rotor-blade fatigue in flapwise bending alone, as extensions to other design cases such as rotor-blade fatigue in edgewise bending as well as fatigue of other wind-turbine components are foreseen.

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DNV

TECHNICAL REPORT

Client :	Danish Energy Agency through Risø National Laboratory
Title :	Reliability-Based Optimization of Design Code for Wind-Turbine Rotor Blades subjected to Fatigue in Flapwise Bending
Report No. :	99-3513

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DNV

REPORT

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Summary

A reliability-based calibration of a design code for wind-turbine rotor blades is presented. The design against fatigue failure in flapwise bending is considered. The calibration is performed by means of a numerical optimization technique in conjunction with probabilistic and deterministic models for load and resistance. The probabilistic models allow for predictions of the reliability against fatigue failure. The deterministic models express design load and design resistance in terms of characteristic values and partial safety factors. The code calibration consists of determination of the partial safety factors such that – over a specified scope of code – designs with reliabilities with minimum deviations from some prescribed target reliability result when applying the deterministic models with this particular set of partial safety factors. The scope of code consists of a number of design cases, each defined as a particular combination of wind turbine, location, and blade material. The set of design cases are selected so as to give the best possible representation of the expected future demand of wind-turbine structures. For tutorial purposes, the scope of code is here limited to the eighteen design cases that are formed by combining three wind turbines, three locations, and two blade materials. Emphasis is placed on the demonstration of the code calibration principles and a proper formulation of the code format.

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		Fatigue
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Title of Report

RELIABILITY-BASED OPTIMIZATION OF DESIGN CODE FOR WIND-TURBINE ROTOR BLADES SUBJECTED TO FATIGUE IN FLAPWISE BENDING

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1. Introduction

A code for design of wind-turbine rotor blades against fatigue failure in flapwise bending is considered. The rotor blades are constructed from fiber-reinforced polyester laminate. The critical location for accumulation of fatigue damage is considered to be at the blade root. The loading consists of a history of bending moment ranges owing to wind exposure and is represented by a model, which has been calibrated from load measurements. The capacity is calculated on the basis of a Miner's rule approach to cumulative damage and capitalizes on a conventional S–N curve formulation.

2. Loading and Response

2.1 Probabilistic model

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the standard deviation σ_U of the wind speed. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \tag{1}$$

in which k and A are site- and height-dependent coefficients. The standard deviation σ_U of the wind speed depends on the 10-minute mean wind speed U_{10}. The distribution of σ_U conditioned on U_{10} can be well represented by a lognormal distribution

$$F_{\sigma_U|U_{10}}(s) = \Phi(\frac{\ln s - h_0}{h_1}) \tag{2}$$

in which $\Phi()$ denotes the standard Gaussian cumulative distribution function, and the coefficients h_0 and h_1 are site-dependent coefficients dependent on U_{10}.

The turbulence intensity I_T is defined as the ratio between the standard deviation σ_U of the wind speed on the one hand, and the 10-minute mean wind speed U_{10} on the other, i.e., $I_T = \sigma_U / U_{10}$. The (U_{10}, I_T) space is referred to in the following. This space is discretized into a number of bins of approximately constant values of U_{10} and I_T.

Note, in particular, that the short-term distribution of I_T conditioned on U_{10}, in conjunction with the long-term distribution of U_{10}, as given in Eqs. (2) and (1), repectively, can be used to calculate the long-term unconditional mean value $E[I_T]$ and standard deviation $D[I_T]$ of the turbulence intensity. These two quantities prove useful in the following for characterization of the wind climate and the loads it exerts on a wind turbine.

One rotor blade is considered in the following. Let X denote the bending moment range at the blade root in flapwise bending. One bending moment range is associated with each load cycle, and load cycles are identified by rain-flow counting. By means of the rain-flow counting technique, observed bending moment histories, each of 10-minute duration, are converted to

10-minute records of the bending moment range X on histogram form. Within each recorded 10-minute interval, also the wind climate parameters U_{10} and I_T are recorded. The (U_{10},I_T) space is discretized in a number of bins as described before. All 10-minute records of X are sorted according to U_{10} and I_T, i.e., each record is localized to a particular bin (U_{10},I_T) in the adopted discretization of the (U_{10},I_T) space. For a particular bin (U_{10},I_T) there will be a total of M 10-minute records of X, and M varies from bin to bin. These M records are used to give an estimate of the short-term distribution of X conditioned on (U_{10},I_T), i.e., $X|(U_{10},I_T)$, on discretized form. The number of load cycles n_{10} in each 10-minute interval is also observed and depends on U_{10} and I_T in manner which for practical purposes herein can be represented as follows

$$n_{10} = d_{0i} + d_{1i}U_{10} + d_{2i}U_{10}^{\;2} + d_{3i}I_T + d_{4i}I_T^{\;2} \tag{3}$$

Because the distribution of $X|(U_{10},I_T)$ is encumbered with uncertainty owing to limited data for its estimation, it is desirable to parametrize the distribution and represent this uncertainty in reliability analyses as uncertainty in the distribution parameters. Such a parametrization is useful also when it is desirable to extrapolate the distribution beyond the range of the available data. The distribution of $X|(U_{10},I_T)$ can be parametrized in terms of its statistical moments. The first three statistical moments are used for this purpose. These moments are the mean value a_1, the standard deviation a_2, and the skewness a_3. Their expected values $E[a_i]$, i=1,2,3, can be estimated based on the observed discretized version of the conditional distribution of $X|(U_{10},I_T)$. Their standard deviations $D[a_i]$, i=1,2,3, and also their correlation matrix ρ can be estimated by a resampling technique such as the jackknife or the bootstrap, see Efron and Tibshirani (1993).

It is assumed that the expected values and standard deviations of a_1, a_2, and a_3 conditioned on U_{10} and I_T are adequately represented by the polynomial surfaces

$$E[a_i] = b_{0i} + b_{1i}U_{10} + b_{2i}U_{10}^{\;2} + b_{3i}I_T + b_{4i}I_T^{\;2} \tag{4}$$

$$D[a_i] = c_{0i} + c_{1i}U_{10} + c_{2i}U_{10}^{\;2} + c_{3i}I_T + c_{4i}I_T^{\;2} \tag{5}$$

in which the coefficients b_{ji} and c_{ji}, j=0,...4, i=1,2,3, are determined by least-squares regressions of all estimated mean values and standard deviations of a_1, a_2, and a_3 over the (U_{10},I_T) space.

Based on the assumption that the central limit theorem holds for the estimates of the three moments a_1, a_2, and a_3, these moments can be represented as

$$a_i = E[a_i] + U_i D[a_i], \qquad i = 1,2,3 \tag{6}$$

in which $\mathbf{U}=(U_1, U_2, U_3)^T$ is a three-dimensional normally distributed variable with zero mean, unit variance, and correlation matrix ρ. Note in this context that U_i is standard notation for a standard normally distributed variable within the field of structural reliability and is not to be confused with any wind speed. Note also that the vector $\mathbf{U}=(U_1, U_2, U_3)^T$ represents the statistical uncertainty in the three moments a_1, a_2, and a_3 owing to the limited data available for their estimation.

A model for the distribution of the bending moment range X is dealt with in the following. Load response amplitudes are often seen to have marginal distributions, which are close to Weibull distributions. The bending moment range is two times such a load response amplitude. Based on the first three moments a_1, a_2, and a_3 of the distribution of the conditional bending moment range $X|(U_{10},I_T)$, this distribution can be modelled as a quadratic expansion of a parent Weibull-distributed variable U_W. The parent Weibull-distributed variable U_W is chosen such that it has the same mean a_1 and the same standard deviation a_2 as the distribution of $X|(U_{10},I_T)$ which is to be modelled. For the case that the skewness of U_W is smaller than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a softening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa(U_W + \xi U_W^{\ 2}) \tag{7}$$

For the case that the skewness of U_W is greater than the skewness a_3 of $X|(U_{10},I_T)$, the quadratic expansion model is a hardening model, and the quadratic expansion reads

$$X = x_{\min} + \kappa \frac{\sqrt{1+4\xi U_W} - 1}{2\xi} \tag{8}$$

In both cases, the model is referred to as a quadratic Weibull model, and the coefficients ξ, κ, and $x_{\min}$ are chosen such that the mean value a_1, the standard deviation a_2, and the skewness a_3 of the distribution of $X|(U_{10},I_T)$ are all preserved. The quadratic Weibull model may be thought of as a generalized or distorted Weibull distribution. Reference is made to Lange and Winterstein (1996).

The section modulus of the rotor blade at the blade root is W, and the stress range S corresponding to the moment range X is $S=X/W$. When a discretization of the stress range space is introduced, then either Eq. (7) or Eq. (8), depending on the value of the skewness a_3, can be applied in conjunction with the distribution function of the parent Weibull variable U_W to calculate the probability content of each interval ΔS of this discretization. The corresponding number of cycles within each such interval in a 10-minute period can be determined as this probability content times the total number of cycles $n_{10}(U_{10},I_T)$ in the 10-minute period. This number can be represented as a deterministic quantity as justified by Ronold et al. (1999).

Integrating contributions from all possible 10-minute wind climate bins (U_{10},I_T), weighted according to the quoted Weibull distribution for U_{10} and the lognormal distribution for I_T, this can be used to establish a distribution of the stress range S over the design life T_L of the rotor blade. Here T_L is taken as 20 years. This compound lifetime distribution of the stress range S is commonly referred to as the 20-year load spectrum and can be expressed in terms of the number of stress cycles n whose associated stress ranges exceed a level S during the design life of the rotor blade. This, in turn, can be used to calculate the number of stress cycles Δn within an interval ΔS of the discretized stress range space.

A model uncertainty is associated with the use of the quadratic Weibull model for representation of the distribution of the bending moment range conditional on the wind climate (U_{10},I_T). Damage predictions by the Miner's sum, as described later and based on such quadratic Weibull distributions for the loading, are therefore multiplied by a random factor F_M. This random factor represents the bias and uncertainty in these damage predictions as associated with the use of the quadratic Weibull model for the conditional load distributions. The distribution type for F_M is taken as a normal distribution. For further details about this model uncertainty factor, reference is made to Ronold et al. (1999).

2.2 Deterministic model

The characteristic moment range distribution is taken as

$$X = k_R X_0 \left(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}\right) \tag{9}$$

in which X denotes the moment range which is exceeded in n stress cycles during the design life T_L, $N_r = f_r T_L$ is the number of rotor cycles in this life, f_r denotes the rotor frequency, X_0 is a characteristic bending moment whose derivation is described below, and k_R is a site- and turbine-specific scaling factor. The value of k_R is taken according to the following formula

$$k_R = 15.997 - 1.8785A + 28.684A \cdot D[I_T] - 275.53D[I_T] + 2.4803E[I_T] - 2.6524R \cdot D[I_T] \tag{10}$$

in which the values of the coefficients have been determined based on data from the nine possible combinations of three wind turbines and three locations. Over this data space, the coefficients have been determined such that the deviations between the cumulative damages as calculated from the median of the compound load distribution, derived as described in Section 2.1, and the cumulative damages as resulting from the characteristic load distribution were minimized. Reference is made to Figure 1 for an example.

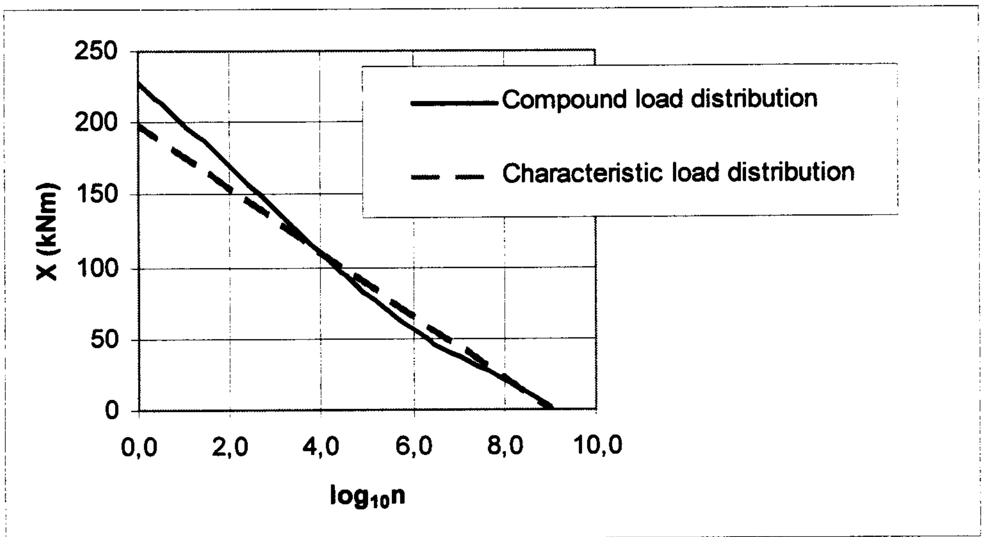

Figure 1 Comparison between data-based compound load distribution and corresponding simplified characteristic load distribution; number of cycles n with bending moment range in excess of X is shown for a 20-year design life

The dependency on A and $E[I_T]$ is in conformance with what is used in the Danish code, while the dependency on $D[I_T]$ is included to improve the fit of the model to data. The dependency on the rotor radius R is included not only to improve the fit of the model to data, but also to properly reflect the amplifying effect on the maximum bending moment owing to the increased effect of the spatial variability of the turbulence field over the rotor blade for increasing R. This effect of the spatial variability of the turbulence field over the rotor blade comes about as the result of an increased ratio between the rotor radius R and the correlation length of the turbulence field, and is not accounted for in the expression for X_0.

Note that the values of A, $E[I_T]$ and $D[I_T]$ are height-dependent and, for each case, need to be calculated for the height of the hub of the particular wind turbine in question.

Note also that the k_R model in Eq. (10) has been established based on a calibration to results for 3 wind turbines on 3 locations, i.e. a total of only 9 combinations of wind turbine and location. Although the achieved model fit is a good fit with only little error, this is a very limited basis for a calibration, and a recalibration and maybe even a revision of the model may therefore be necessary when results from more turbines and more locations become available in the future.

In accordance with the Danish code, see Dansk Ingeniørforening (1992), the characteristic bending moment is defined as

$$X_0 = \frac{\rho}{2} w^2 c C_L \frac{R^2}{3} \tag{11}$$

in which R is the rotor radius measured from the center of the rotor to the tip of the blade, c is characteristic chord length at $2R/3$, C_L is a lift coefficient at $2R/3$, ρ=1.28 kg/m^3 is the density of air, and w is a reference wind speed defined by

$$w^2 = (\frac{4\pi}{3} f_r R)^2 + v_0^2 \tag{12}$$

where f_r is the rotor frequency as before, and v_0 is the 10-minute mean wind speed at stalling of the entire rotor blade.

A load factor γ_f greater than 1.0 is applied as a factor on all load values of the characteristic moment range distribution and the relation $S=X/W$ between stress range S and moment range X is substituted such that the design stress range distribution becomes

$$S = \gamma_f k_R \frac{X_0}{W}(1 - \frac{\log_{10} n}{\log_{10}(3N_r)}) \tag{13}$$

In probabilistic terms, this is recognized as an exponential distribution with cumulative distribution function

$$F_S(s) = 1 - \exp(-\frac{\ln n_{tot} W}{\gamma_f k_R X_0} s) \tag{14}$$

and probability density function

$$f_S(s) = \frac{dF_S(s)}{ds} = \frac{\ln n_{tot} \cdot W}{\gamma_f k_R X_0} \exp(-\frac{\ln n_{tot} \cdot W}{\gamma_f k_R X_0} s) \quad (15)$$

in which $n_{tot}=3N_r=3f_rT_L$ is the design total number of stress cycles in the design life T_L.

3. Capacity

3.1 Probabilistic model

For a given stress range S, the number of cycles N to failure is generally expressed through an S–N curve, $N=BS^{-k}$. However, in tests of composite materials for use in rotor blades, the strain amplitude ε is usually measured rather than the stress range S. Hence, for such materials the number of cycles N to failure is expressed through an ε–N curve. This curve can be expressed by the following relationship

$$\log_{10} N = \log_{10} K - m \log_{10} \varepsilon \quad (16)$$

in which K and m are coefficients. This gives a linear model for $\log_{10}N$

$$\log_{10} N_i = \log_{10} K - m \log_{10} \varepsilon_i + e_i \ , i=1,...n \quad (17)$$

in which the pair ($\log_{10}K$, m) describes the expected behavior and can be estimated by a linear regression analysis based on n observed data pairs (ε_i,N_i). The zero-mean terms e_i denote residuals that represent local variations from test specimen to test specimen, or ideally from one point of the rotor blade to another. The standard deviation σ_e of the residuals e_i will result as a byproduct of the regression analysis, and so will the standard deviations and correlation coefficient of $\log_{10}K$ and m. The distribution of ($\log_{10}K$, m) can be represented as a bivariate normal distribution, and the distribution of the residuals e_i can be represented by another, independent normal distribution. The stress range S that corresponds to the strain amplitude ε can be expressed as $S=2E\varepsilon$, where E denotes the modulus of elasticity of the material in the direction of the loading. The modulus of elasticity is idealized as a constant here.

According to Miner's rule, fatigue failure in a structural material is defined to occur when the cumulative damage D exceeds 1.0, where D is defined as

$$D = \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \quad (18)$$

Here, Δn is the number of load cycles at stress range S in the lifetime of the rotor blade, and N is the number of cycles to failure at this stress range as determined from the S–N curve. The sum is over all stress ranges S_i in a sufficiently fine discretization of the stress range space. When Δn is predicted by the load model, which is described in Section 2.1, then the cumulative damage becomes modified to

$$D = F_M \sum_{i=1}^{k} \frac{\Delta n(S_i)}{N(S_i)} \quad (19)$$

3.2 Deterministic model

For ε–*N* curves, it is a standard approach to select the characteristic ε–*N* curve as the curve that results when the estimated ε-*N* curve is shifted to the left by a distance equal to two times the standard deviation of the residual *e*, hence

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10} \varepsilon - 2\sigma_e \tag{20}$$

Reference is made to Det Norske Veritas (1984) and Dansk Ingeniørforening (1992).

A material factor γ_m greater than 1.0 is introduced. For any given number of cycles to failure, the characteristic strength is divided by this number to give the design strength. Recall the relationship $S=2E\varepsilon$ between stress range and strain amplitude. This implies that the design *S–N* curve becomes

$$\log_{10} N = E[\log_{10} K] - E[m]\log_{10}(\frac{S}{2E}\gamma_m) - 2\sigma_e \tag{21}$$

and this can be reformulated as follows

$$N = K_C(\frac{S}{2E}\gamma_m)^{-E[m]} \tag{22}$$

with $\log_{10}K_C=E[\log_{10}K]-2\sigma_e$. The design damage becomes

$$D_D = \sum_{i=1}^{k}\frac{\Delta n_i}{N_i} = \int_0^{\infty}\frac{n_{tot} f_S(s)ds}{K_C(\frac{S}{2E}\gamma_m)^{-E[m]}} = \frac{n_{tot}}{K_C}\Gamma(E[m]+1)\left(\frac{\gamma_m\gamma_f k_R X_0}{2E\ln n_{tot} W}\right)^{E[m]} \tag{23}$$

in which Γ() denotes the gamma function.

4. Code Calibration

4.1 Limit State Function and Reliability Analysis

The reliability of a particular wind-turbine rotor blade against fatigue failure in flapwise bending can be calculated, once the properties of the rotor blade and wind turbine are specified, and once the location of the wind turbine is given. The properties include the rotor radius *R*, the chord length *c*, the section modulus *W* at the blade root, the rotor frequency f_r, the hub height *z*, and which material the blade is constructed from. The location is characterized by the long-term distribution of the mean wind speed and by the terrain roughness. A first-order reliability method, as described in Madsen et al. (1986), is applied for this purpose. This requires that a limit state function be defined. The critical condition with respect to fatigue failure is considered to be that the cumulative damage over some specified reference period *t* exceeds the capacity of the rotor blade. The limit state function is therefore

$$g(\mathbf{X}) = 1 - D_t(\mathbf{X}) \tag{24}$$

in which D_t denotes the cumulative damage in the reference period t, and where $\mathbf{X}$ denotes the vector of stochastic variables, i.e., $\mathbf{X}=(U_1,U_2,U_3,F_M,m,\log_{10}K,e)^T$. Hence the probability of fatigue failure in the specified reference period t is calculated as

$$P_{F,t} = P[g(\mathbf{X}) < 0] \tag{25}$$

In probabilistic design, the requirement to ensure sufficient structural safety is usually a requirement to the annual probability of failure. For a fatigue problem as the present, where failure is associated with the accumulation of damage over time, the annual probability of failure will increase from one year to the next during the design life. The requirement to the failure probability shall be met in all years during the design life. This implies that if the requirement is fulfilled for the last year during the design life, it will be fulfilled for all years. The design life is taken as 20 years. Two reliability analyses are therefore carried out, one for a reference period of 20 years and another for a reference period of 19 years. The resulting probability of failure in the last year during the design life hence becomes

$$P_F = P_{F,20\text{ years}} - P_{F,19\text{ years}} \tag{26}$$

The result of the reliability analyses is given in terms of the reliability index, which relates to the probability of fatigue failure in the last year of the design life as follows

$$\beta = -\Phi^{-1}(P_F) = -\Phi^{-1}(P_{F,20\text{ years}} - P_{F,19\text{ years}}) \tag{27}$$

The section modulus W at the blade root is used as design parameter. This implies that the rotor blade design can be modified through adjusting W until a satisfactory value of the reliability index β results from the reliability analyses.

4.2 Code Check Function for Deterministic Design

The code check to be performed for deterministic design of a rotor blade against fatigue failure in flapwise bending implies a check that the design damage as calculated from the design stress range distribution in conjunction with the design S–N curve shall be less than 1.0. This implies a check that the following requirement is fulfilled

$$1 - D_D \geq 0 \tag{28}$$

The left-hand side of this inequality is known as the code check function h. The inequality itself is also known as the design rule. Note that when the inequality is turned into an equation, this equation implies the design equation, which is the so-called code check limit. The code check function will be used later and is therefore quoted explicitly here as

$$h = 1 - \frac{n_{tot}}{K_C}\Gamma(E[m]+1)\left(\frac{\gamma_m \gamma_f k_R X_0}{2E \ln n_{tot} W}\right)^{E[m]} \tag{29}$$

4.3 Scope of code

The scope of a design code is by definition the class of structures for which the code is meant to apply. This can be any set of structures of a particular type whose design is governed by one or more limit states. The scope of code is a parameterized set of structures. Here, the scope of code is limited to wind-turbine rotor blades whose design is governed by fatigue failure in flapwise bending. For calibration of partial safety factors for use in conventional deterministic design, the scope of code is represented by a number of design cases. These design cases are chosen such that they reflect the expected future demand of structures within the class of structures covered by the scope of code. A design case is formed by a particular combination of a wind turbine, a location, and a rotor blade material. Here, a wind turbine is defined as a turbine of a specific make and equipped with a specific rotor blade. A location is a site with a specific wind climate and terrain roughness. A material is taken as a fiber-reinforced polyester laminate with a particular *S–N* curve. In this example, three different wind turbines are considered in conjunction with three locations and two materials. This allows for definition of a scope of code with up to 18 design cases. These design cases are listed in Table 1. The 18 design cases as listed in Table 1 form a full 3×3×2 scope matrix. Table 2 lists quantities, which are assigned the same fixed values for all of the design cases. Tables 3 through 6 give the particular properties of the three wind turbines and their respective load distributions.

As described in a previous section, each wind turbine is characterized by the distribution of the flapwise bending moment at the blade root. This distribution is determined from a set of conditional distributions, which are conditioned on the wind climate (U_{10},I_T) and obtained from measurements on the turbine as installed on a specific location. These observed conditional distributions are here used for the same turbine installed also at other locations than the one where the measurements were obtained. Such a use really requires that the rotor blades for given (U_{10},I_T) will experience the same autocorrelation function of the wind turbulence field over the blade, regardless of location. This assumption cannot always be expected to be fulfilled. However, it will still be a reasonable assumption for the purpose of a code calibration as the present, as long as the resulting design cases can be assumed to be representative of the class of structures to be covered by the code.

The three locations are distinguished by the terrain roughness parameter and the wind climate. The terrain roughness is represented by the terrain roughness parameter z_0. The wind climate is represented by the scale parameter A_{25} of the long-term Weibull distribution of the 10-minute mean wind speed at a height 25 m above the ground, and by the distribution of the standard deviation σ_U of the wind speed conditioned on the 10-minute mean wind speed. The scale parameter A of the distribution of the 10-minute mean wind speed at the hub height z of the wind turbine is needed in the analysis and is calculated as follows

$$A = A_{25} \frac{\ln \frac{z}{z_0}}{\ln \frac{25}{z_0}} \qquad (30)$$

in which z and z_0 are given in m. Table 7 gives location-specific data. Table 8 gives expressions for the coefficients h_0 and h_1 that enter into the formula for the distribution of the standard deviation σ_U of the wind speed conditioned on the 10-minute mean wind speed U_{10}, cfr. Eq. (2). These expressions have been obtained by fits to available data from wind measurements at the three locations. The measurements have been obtained at heights of between 35 and 45 m, varying from one location to the next. The hub heights of the three wind turbines vary within the same range. For practical purposes, it is assumed that the distribution of σ_U conditioned on U_{10} at a specific location, as determined from the wind measurements, can be applied to all three wind turbines, even when the hub is not located at exactly the same height as the one at which the wind measurements were obtained. This implies that all height dependency of wind speed is modelled through the height dependency of the scale parameter A in the distribution of U_{10}.

The two materials have identical mean properties (m,$\log_{10}K$) as quoted in Table 2 and are only distinguished by having different variability about these mean properties, expressed in terms of a different standard deviation σ_e. For material #1 σ_e=0.396, and for material #2 σ_e=0.3.

A scope of code consisting of all 18 defined design cases is used in the following.

Table 1 Design cases

Design case	Wind turbine	Location	Material	Design case	Wind turbine	Location	Material
1	1	1	1	10	1	1	2
2	1	2	1	11	1	2	2
3	1	3	1	12	1	3	2
4	2	1	1	13	2	1	2
5	2	2	1	14	2	2	2
6	2	3	1	15	2	3	2
7	3	1	1	16	3	1	2
8	3	2	1	17	3	2	2
9	3	3	1	18	3	3	2

Table 2 Quantities, common for all design cases

Quantity	Description	Value
T_L (years)	Design life	20
C_L	Lift coefficient at $2R/3$	1.5
ρ (kg/m^3)	Density of air	1.028
E (kPa)	Young's modulus	$29.7 \cdot 10^6$
$E[m]$	S–N curve slope, mean	7.912
$D[m]$	S–N curve slope, st.dev.	0.247
$E[\log_{10}K]$	S–N curve intercept, mean	−12.372
$D[\log_{10}K]$	S–N curve intercept, st.dev.	0.513
$\rho_{m,\log10K}$	S–N curve correlation coefficient	−0.996
k	Slope parameter of long-term Weibull distribution for wind speed U_{10}	1.9

Table 3 Wind turbines

Quantity	Description	Value for turbine #1	Value for turbine #2	Value for turbine #3
R (m)	Radius of rotor	20.5	21.5	17.5
z (m)	Hub height of rotor	35.0	44.0	35.0
c (m)	Characteristic cord length at $2R/3$	1.18	1.00	0.86
f_r (sec^{-1})	Rotor frequency	0.5033	0.4800	0.5833
N_r	No. of rotations	$0.317 \cdot 10^9$	$0.303 \cdot 10^9$	$0.368 \cdot 10^9$
n_{tot}*	Design cycles	$0.951 \cdot 10^9$	$0.909 \cdot 10^9$	$1.104 \cdot 10^9$
v_0 (m/sec)	Wind speed at stalling	14.6	14.6	14.6
m_{10}	No. of 10-minute load data series recorded	1183	554	891
$E[F_M]$	Model uncertainty factor, mean	0.831	0.855	0.989
$D[F_M]$	Model uncertainty factor, st.dev.	0.221	0.184	0.258

*) derived from other quantities

Table 4 Estimated Coefficients in Polynomial Models for $E[a_i]$ and $D[a_i]$ for Bending Moment Ranges

Turbine	i	b_0	b_1	b_2	b_3	b_4	c_0	c_1	c_2	c_3	c_4
1	1	-5.057	1.555	-0.0092	244.1	-575.1	2.251	-0.086	0.0069	-20.74	72.46
	2	4.688	0.727	0.0101	228.5	-551.2	2.002	0.059	-0.0026	-20.75	64.38
	3	1.639	0.025	-0.0035	1.677	-5.784	0.082	0.011	-0.0006	-1.193	3.461
2	1	29.96	-3.457	0.1843	4.655	904.6	0.063	-0.455	0.0177	46.02	-75.87
	2	34.39	-3.544	0.1721	3.815	797.7	-0.400	-0.456	0.0164	63.35	-181.7
	3	0.824	0.056	-0.0026	7.069	-31.01	0.184	-0.038	0.0014	1.524	-4.479
3	1	2.190	0.363	0.0199	-2.813	54.35	0.743	-0.063	0.0036	-9.012	52.538
	2	5.273	-0.169	0.0473	-4.988	47.38	0.973	-0.083	0.0044	-11.04	62.162
	3	2.786	-0.268	0.0097	-10.66	49.13	0.284	-0.027	0.0011	-2.721	17.686

Units of coefficients are consistent with bending moment ranges quoted in units of kNm and wind speeds in m/sec.

Table 5 Correlation matrix ρ for $(a_1,a_2,a_3)^T$

	Turbine 1	Turbine 2	Turbine 3
ρ	$\begin{bmatrix} 1.000 & 0.937 & -0.346 \\ 0.937 & 1.000 & -0.210 \\ -0.346 & -0.210 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.857 & -0.110 \\ 0.857 & 1.000 & 0.002 \\ -0.110 & 0.002 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.901 & -0.310 \\ 0.901 & 1.000 & -0.037 \\ -0.310 & -0.037 & 1.000 \end{bmatrix}$

Table 6 Estimated Coefficients in Polynomial Model for n_{10}

Turbine	d_0	d_1	d_2	d_3	d_4
1	1703.7	−102.83	7.027	−10.08	1402
2	37.797	234.46	−6.510	−6013.1	26733
3	2006.7	−89.517	4.946	4624.9	−30089

Table 7 Location-specific data

Quantity	Description	Location #1	Location #2	Location #3
z_0 (m)	Terrain roughness parameter	0.0806	0.0238	0.0487
A_{25} (m/sec)	Scale parameter for wind speed	7.33	8.13	7.73
$E[I_T]$	Mean turbulence intensity at 25 m height	0.139	0.087	0.115
$D[I_T]$	St.dev. of turbulence intensity at 25 m height	0.094	0.074	0.055

Table 8 Coefficients h_0 and h_1 for distribution of standard deviation σ_U of wind speed

Location	h_0 (m/sec)	h_1^2 (m^2/sec^2)
#1	$-3.1530+1.3793\cdot\ln(U_{10}+1.7434)$	$0.0051733+0.40467\cdot\exp(-0.19798\cdot U_{10})$
#2	$-1.7946+0.16157\cdot\ln U_{10}+0.099541\cdot U_{10}$	$3.9890\cdot\exp(-0.25880\cdot U_{10})$ $-3.8486\cdot\exp(-0.29306\cdot U_{10})$
#3	$-3.250+1.7957\cdot\ln U_{10}-0.062711\cdot U_{10}$	$8.9070\cdot\exp(-0.46932\cdot U_{10})$ $-9.9136\cdot\exp(-0.56836\cdot U_{10})$ for $U_{10}>3$ m/sec 0.614^2 for $U_{10}\leq 3$ m/sec

4.4 Target reliability

The code objective is a specified target reliability index over the specified set of design cases, which are chosen as representative for the scope of code. The choice for the target reliability index can be derived from a risk acceptance criterion, or by requiring that the safety level as resulting from a design by a structural reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society. Here, as an example, consider a design situation where brittle failure of a non-redundant structure is at stage and has less serious consequence. This would be a reasonable classification for a rotor blade design against fatigue where human life is at negligible risk. According to Nordic Committee on Building Regulations (1978) and Det Norske Veritas (1992), the requirement to the annual failure probability for design under such a classification is 10^{-5}. The corresponding target reliability index is β_T=4.265, and the most critical year with respect to fatigue during the 20-year design life is the last year.

4.5 Code Philosophy, Penalty Function, and Code Optimization

When carrying out designs according to the code, deviations in the achieved safety level from the target safety level are undesirable and are therefore penalized. For a particular design case, a penalty function is defined

$$p = k(\beta - \beta_T) + \exp(-k(\beta - \beta_T)) - 1 \quad (31)$$

in which β is the achieved reliability index and β_T is the target reliability index. This penalty function penalizes under-design more than over-design, and the coefficient k governs to what extent under-design is penalized more than over-design. Reference is made to Figure 2 and to

Lind (1977). In this study, the value of k is set to 1.5, and the sensitivity to this choice is investigated.

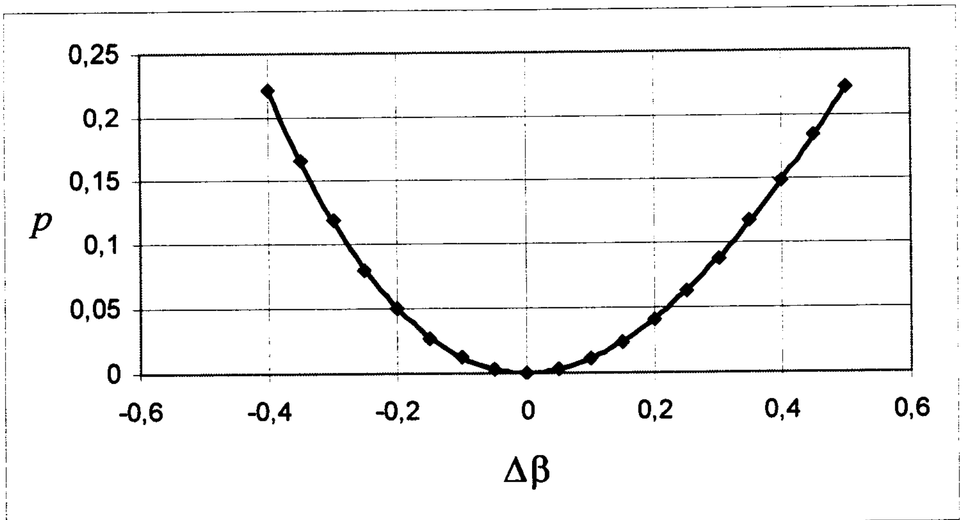

Figure 2 Lind's penalty function

The code philosophy implies that the code check function shall be greater than or equal to zero for all design cases included in the scope of code, and the code check limit shall be fulfilled for at least one of the design cases. Furthermore, the code philosophy implies that the scatter of the resulting reliabilities by the code about the prescribed target reliability shall be minimal, i.e., the weighted sum of penalties over the four design cases that represent the scope of code shall be the least possible. Therefore, the optimal set of partial safety factors (γ_f,γ_m) is achieved as the solution to the following optimization problem

$$\min_{W_i,\gamma_f,\gamma_m} \sum_{i=1}^{I} w_i p(\beta(U_{1i},U_{2i},U_{3i},F_M,m,\log_{10} K,e,W_{ij}) - \beta_T) \tag{32}$$

subject to $h_i(W_i,\gamma_f,\gamma_m) \geq 0, i=1,I$, and $\prod_{i=1}^{I} h_i(W_i,\gamma_f,\gamma_m) = 0$.

Reference is made to Hauge et al. (1992). The weighting factors w_i, i=1,I, are here set equal to 1/I under the assumption of an equal demand in the future of the I design cases that represent the scope of code.

4.6 Analysis Tool and Input File

The computer program PROCODE for reliability-based code calibration is used for the code optimization described above. PROCODE is a development version of the general-purpose structural reliability analysis program PROBAN, see Det Norske Veritas (1993, 1997). The input file used for the present wind turbine problem is reproduced in Appendix A.

4.7 Code Format and Results of Code Optimization

It appears from the expression for the code check function that the result of the code optimization will come out as a requirement to the product of the load factor and the material factor. The simplest code format, which can be adopted, implies that the two partial safety factors γ_f and γ_m are taken as constants over the scope of code, independent of properties that may vary from design case to design case within the scope of code. In this case, the result of the code optimization will thus in practice be a single constant "old-fashioned" safety factor to be applied to all designs. However, it turns out that if such a simple code format is chosen, then the optimization of the safety factor product over the scope of code will yield a considerable scatter in the achieved reliability indices β_i, i=1,I, over the scope of code. The deviations from the target reliability index β_T will be of the order ±1.0, which is rather significant. This is unacceptable and is not a desirable result, as the target reliability has not been met closely enough. In other words, the safety level achieved over the scope of code is not sufficiently uniform. This indicates that the formulation of the design code is not sufficient to cover the entire scope of code, or the scope of code is too wide to be adequately covered by the present code formulation. This calls for an improvement of the code formulation, as it is not an option to reduce the scope of code.

For such an improvement it is of interest to consider refinements of each of the two partial safety factors separately. This can be done with a view to the uncertainties that these factors are meant to cover and how these uncertainties can be expected to vary over the scope of code.

For load, the uncertainty is a statistical uncertainty associated with limited data for determination of the load distributions. Data are available in terms of a number of 10-minute series of load ranges. The more 10-minute series, the smaller is the statistical uncertainty in the interpreted load distributions. Over the scope of code, this uncertainty varies from design case to design case, because the available number of 10-minute data series is different for the different wind turbines, which have been chosen to represent the scope of code. For the purpose of determining load distributions, the number m_{10} of 10-minute series is thus not only a measure of the data quantity but also an indicator of the level of statistical uncertainty and thus the quality of the estimated load distributions. It may be reasonable to formulate the load factor as a function of m_{10} so as to penalize more, by a higher γ_f, for fewer data and poorer data quality. The load distributions are expressed in terms of the wind speed conditions at the hub, regardless of what happens away from the hub. The longer the rotor blade, the larger is the ratio between the rotor radius R and the correlation length of the wind turbulence field, and the wider will the load distributions at the blade root be. This is so because of the associated stronger influence of the spatial variability of the wind turbulence field when R increases. One implication of this is that the larger the value of R, the higher is the number of 10-minute series, m_{10}, required to reach a particular accuracy of the determination of the load distributions at the blade root. On this background it is not unreasonable to expect the uncertainty associated with the determination of the load distributions to be larger when the rotor blades are longer, and it may be reasonable to let γ_f be a function also of the rotor radius R. Therefore, the following format for the load factor γ_f is adopted

$$\gamma_f = \gamma_0 + \gamma_1 \frac{R^{\gamma_2}}{m_{10}^{\gamma_3}} \tag{33}$$

For the fatigue capacity as embedded in the *S–N* curve, the natural variability about the expected *S–N* curve is found to be the major uncertainty source. This uncertainty source is characterized by the standard deviation σ_e of the residuals of $\log_{10}N$ about the expected value. Accordingly, the following format for the material factor γ_m is adopted

$$\gamma_m = 1.0 + \gamma_4 \sigma_e^{\gamma_5} \tag{34}$$

Table 8 Results of Code Optimization

Partial safety factor	Optimized safety factor	
γ_f	$0.998 + 0.0589 \dfrac{R^{0.956}}{m_{10}^{0.250}}$	
γ_m	$1.0 + 0.438\sigma_e^{1.444}$	
Design case	Achieved reliability index	Scatter (deviation from target)
1	4.281	0.016
2	4.367	0.102
3	4.255	–0.010
4	4.248	–0.017
5	4.192	–0.073
6	4.367	0.102
7	4.223	–0.042
8	4.205	–0.060
9	4.257	–0.008
10	4.320	0.055
11	4.427	0.162
12	4.277	0.012
13	4.188	–0.077
14	4.124	–0.141
15	4.350	0.085
16	4.220	–0.045
17	4.218	–0.047
18	4.292	0.027

With the more detailed format of the partial safety factors, the code optimization is carried through for the scope of code that consists of the eighteen design cases numbered (1-18). The results are presented in Table 8, and it appears that – overall – a moderate scatter of the achieved reliability indices about the target reliability index results. This is a satisfactory result, which may indicate that the refined code format is adequate for the selected scope of code.

To get an idea of which partial safety factors are actually implied by the obtained result of the code optimization, substitution of relevant values for R, m_{10} and σ_e as given in Section 4.3 yields a load factor γ_f in the range 1.16-1.23 and a material factor γ_m in the range 1.07-1.12. None of these ranges are overly wide, and one could consider replacing each of them with a fixed value to be used for all designs. This would be practical. However, this would bring us back to the simple code format, which was investigated initially, and which resulted in a too large scatter of the reliability index β. One may deduce from this that β, in fact, is quite sensitive to even small changes in the partial safety factors, and that the refined code format with the detailed expressions for the partial safety factors is needed to maintain a sufficiently uniform safety level in the designs that result from use of the code.

The format for γ_f was chosen so as to honor the designer for the quality of his data, namely by letting the available number of 10-minute series of load measurements be a measure of the accuracy of the load distributions and letting γ_f be a decreasing function of m_{10}. In addition, γ_f was made dependent on the rotor radius R as supported by the wind turbine data available for the present study. The available number of 10-minute series of load measurements in the high wind-speed range m_h may be another measure of the load data quality, since the high wind-speed range, here defined as u_{10}>15 m/s, is of great importance for damage accumulation. A possibility for refinement of the representation of the data quality in the expression for γ_f has been investigated by modifying the format for γ_f to become a decreasing function of m_h. However, based on the available wind-turbine data, such a refinement did not prove useful, as the dependency of γ_f on m_h comes out as an increasing function rather than as the sought-after decreasing function. It was as if the data quality is sufficiently reflected by the modelled dependency on m_{10} alone.

As stated in a previous section, the requirement to the partial safety factors γ_f and γ_m in the present code optimization problem is a requirement to their product $\gamma_m \times \gamma_f$. This implies that one is free to linearly resize γ_f and γ_m as long as their product is kept unchanged. As all uncertainty in the load distribution is epistemic uncertainty, and as the characteristic load distribution is taken close to a "median distribution" with respect to this uncertainty, it would be reasonable to let γ_f take on a value 1.0 when the amount of data goes towards infinity. Hence, the following set of partial safety factors would be a good choice,

$$\gamma_f = 1.0 + 0.0590 \frac{R^{0.956}}{m_{10}^{0.250}} \tag{35}$$

and

$$\gamma_m = 0.998 + 0.437 \sigma_e^{1.444} \tag{36}$$

The code optimization is carried out under application of Lind's penalty function with the coefficient k set equal to 1.5. When assessing under- and overdesign on a reliability index scale, this choice for k, which is a somewhat subjective choice, leads to a slightly higher penalty for underdesign than for overdesign. Other values for k would reflect other choices for how much to penalize underdesign relative to overdesign. However, when the scatter in the achieved reliability indices β about the target β_T is as limited as indicated in Table 8, the sensitivity in the resulting partial safety factors will be minor for moderate changes in k. This is in agreement with results reported by Ronold (1999) for optimization of a design code for a class of structures encountered in the offshore industry.

5. Discussion and conclusions

An example of a code optimization for wind turbines has been worked out and has been presented. A code for the design of rotor blades against fatigue failure in flapwise bending has been chosen as the subject for the example. The example has been based on data for three wind turbines, three locations, and two materials. A code format with two partial safety factors has been considered. A load factor has been applied to all stress amplitudes of the characteristic load distribution, which has been defined, and a material factor has been applied to the stress terms in the characteristic *S–N* curve, which has been defined and which governs the capacity.
This code format has been subject to some further refinement. The load factor has been expressed as a function of the following two quantities

- the number of available 10-minute series of load measurement data, which serves as a measure of the data quality for the load distribution of a particular wind turbine
- the rotor radius, which appears as a parameter well suited to further explain load distribution uncertainty as observed for the wind turbines considered here

The material factor has in an analogous way been expressed as a function of the standard deviation of the residuals of the *S–N* curve as this is a direct measure of the natural variability of the fatigue life.

A scope of code has been formed, consisting of a set of design cases obtained as the 18 possible combinations of the three wind turbines, the three locations, and the two blade materials. This scope has in this study been assumed to be representative for the future demand. A target reliability index β=4.265 has been chosen, corresponding to a target annual probability of failure of 10^{-5}. A penalty function for penalizing deviations from this target has been selected according to a formula given in the literature.

For fatigue, failure during the last year of the design life is most critical, and failure during the last year of a 20-year design life has been considered. The code optimization has been carried out for the selected 18 design cases, and this has been done such that the selected penalty function has become minimized. The resulting optimal values of the coefficients in the functional expressions for the two partial safety factors have been presented. It has been demonstrated how the reliability indices come out for the 18 design cases and how they are rather moderately scattered about the target value of 4.265. The achieved reliability indices thus indicate that a fairly uniform safety level results from designs by means of the optimized set of partial safety factors.

The scope of code has in this study been limited to a set of design cases defined from only three wind turbines and three locations. The three wind turbines included in the scope consisted of turbines with rated powers in the range 450-600 kW as achieved by hub heights of 35-44 m and rotor diameters of 35-43 m. Current trends within the wind turbine industry point in the direction of larger turbines with rated powers of up to 2 MW. For future updating and refinement of the code optimization presented in this report, it should therefore be considered to include a number of more wind turbines in the scope of code, preferably turbines of higher rated powers than those of the currently included turbines. This will be possible as load measurements from large turbines become available.

The three locations used for definition of the scope of code represent three different wind climates. These three locations need not necessarily be sufficient to represent all possibilities which can be expected for future locations of wind turbines. The mere presence of an obstruction, such as a house nearby a prospective wind turbine site, is sufficient to significantly alter the local wind climate and introduce additional turbulence. This implies that a much larger number of typical locations may be required to adequately cover the future demand. A future update of the presented code optimization should therefore also include a number of more locations and corresponding wind climates in the scope of code. This will be possible when more data from wind speed measurements become available from a wider selection of typical locations.

6. References

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Appendix A: PROCODE input file

```
% Fatigue of wind-turbine rotor blade:
%
% Define safety factor parameters and variables
%
CREATE VARIABLE Safe00 'Safety factor for load'     FIXED 1.0
CREATE VARIABLE Safe01 'Safety factor for load'     FIXED 1.0
CREATE VARIABLE Safe02 'Safety factor for load'     FIXED 1.0
CREATE VARIABLE Safe03 'Safety factor for load'     FIXED 1.0
CREATE VARIABLE Safe04 'Safety factor for strength' FIXED 1.0
CREATE VARIABLE Safe05 'Safety factor for strength' FIXED 1.0
CREATE VARIABLE Safe06 'Safety factor for strength' FIXED 1.0
DEFINE SAFETY-FACTOR ADD SafeL0 'Safety factor for load'     Safe00
DEFINE SAFETY-FACTOR ADD SafeL1 'Safety factor for load'     Safe01
DEFINE SAFETY-FACTOR ADD SafeL2 'Safety factor for load'     Safe02
DEFINE SAFETY-FACTOR ADD SafeL3 'Safety factor for load'     Safe03
DEFINE SAFETY-FACTOR ADD SafeS4 'Safety factor for strength' Safe04
DEFINE SAFETY-FACTOR ADD SafeS5 'Safety factor for strength' Safe05
DEFINE SAFETY-FACTOR ADD SafeS6 'Safety factor for strength' Safe06
CREATE VARIABLE SafeL0 'Safety factor for load'     GENERIC SAFETY-FACTOR SafeL0
CREATE VARIABLE SafeL1 'Safety factor for load'     GENERIC SAFETY-FACTOR SafeL1
CREATE VARIABLE SafeL2 'Safety factor for load'     GENERIC SAFETY-FACTOR SafeL2
CREATE VARIABLE SafeL3 'Safety factor for load'     GENERIC SAFETY-FACTOR SafeL3
CREATE VARIABLE SafeS4 'Safety factor for strength' GENERIC SAFETY-FACTOR SafeS4
CREATE VARIABLE SafeS5 'Safety factor for strength' GENERIC SAFETY-FACTOR SafeS5
CREATE VARIABLE SafeS6 'Safety factor for strength' GENERIC SAFETY-FACTOR SafeS6
%
% Define design parameter names and variables:
%
CREATE VARIABLE w 'Section modulus' FIXED 1.0
DEFINE DESIGN-PARAMETER ADD w 'Section modulus' w
CREATE VARIABLE ws 'Despar' GENERIC DESIGN-PARAMETER w
%
% Define design situation parameter names and variables:
%
DEFINE DESIGN-SITUATION-PARAMETER ADD a0 'Response surface coeff 1'
DEFINE DESIGN-SITUATION-PARAMETER ADD a1 'Response surface coeff 2'
DEFINE DESIGN-SITUATION-PARAMETER ADD a2 'Response surface coeff 3'
DEFINE DESIGN-SITUATION-PARAMETER ADD x0 'Char bending moment range'
DEFINE DESIGN-SITUATION-PARAMETER ADD ntot 'Design total cycle number'
DEFINE DESIGN-SITUATION-PARAMETER ADD n10 'Design total cycle number'
DEFINE DESIGN-SITUATION-PARAMETER ADD rl 'Length of rotor blade'
CREATE VARIABLE a0 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER a0
CREATE VARIABLE a1 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER a1
CREATE VARIABLE a2 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER a2
CREATE VARIABLE x0 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER x0
CREATE VARIABLE ntot 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER ntot
CREATE VARIABLE n10 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER n10
CREATE VARIABLE rl 'Dessitpar' GENERIC DESIGN-SITUATION-PARAMETER rl
%
% Define environmental condition parameter names and variables:
%
DEFINE ENVIRONMENTAL-CONDITION-PARAMETER ADD kc 'S-N curve intercept'
DEFINE ENVIRONMENTAL-CONDITION-PARAMETER ADD sigmae 'Stdev of residuals'
CREATE VARIABLE kc 'Envconpar' GENERIC ENVIRONMENTAL-CONDITION-PARAMETER kc
CREATE VARIABLE sigmae 'Envconpar' GENERIC ENVIRONMENTAL-CONDITION-PARAMETER
    sigmae
%
CREATE VARIABLE u ' ' DISTRIBUTION Normal Mean-StD 0. 1.
CREATE VARIABLE m ' ' FIXED 7.912
CREATE VARIABLE gammam ' ' FIXED 33412.0
CREATE VARIABLE emodul ' ' FIXED 29700000.0
CREATE VARIABLE d1 ' ' FUNCTION Product ( ONLY a1 ws )
CREATE VARIABLE d2 ' ' FUNCTION Product ( ONLY a2 ws ws )
CREATE VARIABLE g ' ' FUNCTION Sum ( ONLY u a0 d1 d2 )
CREATE EVENT fail ' ' SINGLE g < 0.0
%
% Design situations:
```

```
%
CREATE VARIABLE a0_1 'Coeff 1. Design situation 1.' FIXED -4.4209
CREATE VARIABLE a0_2 'Coeff 1. Design situation 2.' FIXED -4.3869
CREATE VARIABLE a0_3 'Coeff 1. Design situation 3.' FIXED -4.4833
CREATE VARIABLE a0_4 'Coeff 1. Design situation 4.' FIXED -3.4330
CREATE VARIABLE a0_5 'Coeff 1. Design situation 5.' FIXED -3.5886
CREATE VARIABLE a0_6 'Coeff 1. Design situation 6.' FIXED -3.7259
CREATE VARIABLE a0_7 'Coeff 1. Design situation 7.' FIXED -3.6977
CREATE VARIABLE a0_8 'Coeff 1. Design situation 8.' FIXED -4.1182
CREATE VARIABLE a0_9 'Coeff 1. Design situation 9.' FIXED -3.8572
CREATE VARIABLE a0_10 'Coeff 1. Design situation 10.' FIXED -6.5779
CREATE VARIABLE a0_11 'Coeff 1. Design situation 11.' FIXED -6.9154
CREATE VARIABLE a0_12 'Coeff 1. Design situation 12.' FIXED -6.5442
CREATE VARIABLE a0_13 'Coeff 1. Design situation 13.' FIXED -4.8275
CREATE VARIABLE a0_14 'Coeff 1. Design situation 14.' FIXED -4.9619
CREATE VARIABLE a0_15 'Coeff 1. Design situation 15.' FIXED -5.5127
CREATE VARIABLE a0_16 'Coeff 1. Design situation 16.' FIXED -5.9642
CREATE VARIABLE a0_17 'Coeff 1. Design situation 17.' FIXED -6.0454
CREATE VARIABLE a0_18 'Coeff 1. Design situation 18.' FIXED -5.3984
CREATE VARIABLE a1_1 'Coeff 2. Design situation 1.' FIXED 6899.0
CREATE VARIABLE a1_2 'Coeff 2. Design situation 2.' FIXED 7053.0
CREATE VARIABLE a1_3 'Coeff 2. Design situation 3.' FIXED 7991.0
CREATE VARIABLE a1_4 'Coeff 2. Design situation 4.' FIXED 5074.0
CREATE VARIABLE a1_5 'Coeff 2. Design situation 5.' FIXED 5851.0
CREATE VARIABLE a1_6 'Coeff 2. Design situation 6.' FIXED 7423.0
CREATE VARIABLE a1_7 'Coeff 2. Design situation 7.' FIXED 18470.8
CREATE VARIABLE a1_8 'Coeff 2. Design situation 8.' FIXED 17939.6
CREATE VARIABLE a1_9 'Coeff 2. Design situation 9.' FIXED 17912.5
CREATE VARIABLE a1_10 'Coeff 2. Design situation 10.' FIXED 9386.
CREATE VARIABLE a1_11 'Coeff 2. Design situation 11.' FIXED 10149.
CREATE VARIABLE a1_12 'Coeff 2. Design situation 12.' FIXED 10719.
CREATE VARIABLE a1_13 'Coeff 2. Design situation 13.' FIXED 6425.5
CREATE VARIABLE a1_14 'Coeff 2. Design situation 14.' FIXED 7334.
CREATE VARIABLE a1_15 'Coeff 2. Design situation 15.' FIXED 9893.
CREATE VARIABLE a1_16 'Coeff 2. Design situation 16.' FIXED 26456.3
CREATE VARIABLE a1_17 'Coeff 2. Design situation 17.' FIXED 23756.3
CREATE VARIABLE a1_18 'Coeff 2. Design situation 18.' FIXED 22193.8
CREATE VARIABLE a2_1 'Coeff 3. Design situation 1.' FIXED -0.86000e+06
CREATE VARIABLE a2_2 'Coeff 3. Design situation 2.' FIXED -0.90000e+06
CREATE VARIABLE a2_3 'Coeff 3. Design situation 3.' FIXED -0.11800e+07
CREATE VARIABLE a2_4 'Coeff 3. Design situation 4.' FIXED -0.50500e+06
CREATE VARIABLE a2_5 'Coeff 3. Design situation 5.' FIXED -0.67000e+06
CREATE VARIABLE a2_6 'Coeff 3. Design situation 6.' FIXED -0.10600e+07
CREATE VARIABLE a2_7 'Coeff 3. Design situation 7.' FIXED -5.381944e+06
CREATE VARIABLE a2_8 'Coeff 3. Design situation 8.' FIXED -5.468750e+06
CREATE VARIABLE a2_9 'Coeff 3. Design situation 9.' FIXED -4.861111e+06
CREATE VARIABLE a2_10 'Coeff 3. Design situation 10.' FIXED -0.12400e+07
CREATE VARIABLE a2_11 'Coeff 3. Design situation 11.' FIXED -0.15000e+07
CREATE VARIABLE a2_12 'Coeff 3. Design situation 12.' FIXED -0.16600e+07
CREATE VARIABLE a2_13 'Coeff 3. Design situation 13.' FIXED -0.65500e+06
CREATE VARIABLE a2_14 'Coeff 3. Design situation 14.' FIXED -0.84000e+06
CREATE VARIABLE a2_15 'Coeff 3. Design situation 15.' FIXED -0.15400e+07
CREATE VARIABLE a2_16 'Coeff 3. Design situation 16.' FIXED -9.288194e+06
CREATE VARIABLE a2_17 'Coeff 3. Design situation 17.' FIXED -7.204861e+06
CREATE VARIABLE a2_18 'Coeff 3. Design situation 18.' FIXED -4.427083e+06
%Simple kR model - updated Sept 1999 - 6 coeffs - linear in A, EIt, DIt,
%ADIt, RDIt
CREATE VARIABLE x0_1 'Turbine 1. Design situation 1,10.' FIXED 624.89
CREATE VARIABLE x0_2 'Turbine 2. Design situation 2,11.' FIXED 616.34
CREATE VARIABLE x0_3 'Turbine 3. Design situation 3,12.' FIXED 546.53
CREATE VARIABLE x0_4 'Turbine 1. Design situation 4,13.' FIXED 713.89
CREATE VARIABLE x0_5 'Turbine 2. Design situation 5,14.' FIXED 629.08
CREATE VARIABLE x0_6 'Turbine 3. Design situation 6,15.' FIXED 519.30
CREATE VARIABLE x0_7 'Turbine 1. Design situation 7,16.' FIXED 200.28
CREATE VARIABLE x0_8 'Turbine 2. Design situation 8,17.' FIXED 223.17
CREATE VARIABLE x0_9 'Turbine 3. Design situation 9,18.' FIXED 210.88
CREATE VARIABLE ntot_1 'Turbine 1. Design situation 1-3,10-12.' FIXED 951.e+06
```

```
CREATE VARIABLE ntot_2 'Turbine 2. Design situation 4-6,13-15.' FIXED 909.e+06
CREATE VARIABLE ntot_3 'Turbine 3. Design situation 7-9,16-18.' FIXED 1104.e+06
CREATE VARIABLE n10_1 'Turbine 1. Design situation 1-3,10-12.' FIXED 1183.
CREATE VARIABLE n10_2 'Turbine 2. Design situation 4-6,13-15.' FIXED 554.
CREATE VARIABLE n10_3 'Turbine 3. Design situation 7-9,16-18.' FIXED 891.
CREATE VARIABLE rl_1 'Turbine 1. Design situation 1-3,10-12.' FIXED 20.5
CREATE VARIABLE rl_2 'Turbine 2. Design situation 4-6,13-15.' FIXED 21.5
CREATE VARIABLE rl_3 'Turbine 3. Design situation 7-9,16-18.' FIXED 17.5
CREATE DESIGN-SITUATION DESSIT_1 'Design Situation 1' ( ONLY a0 a0_1 a1 a1_1
    a2 a2_1 x0 x0_1 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_2 'Design Situation 2' ( ONLY a0 a0_2 a1 a1_2
    a2 a2_2 x0 x0_2 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_3 'Design Situation 3' ( ONLY a0 a0_3 a1 a1_3
    a2 a2_3 x0 x0_3 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_4 'Design Situation 4' ( ONLY a0 a0_4 a1 a1_4
    a2 a2_4 x0 x0_4 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_5 'Design Situation 5' ( ONLY a0 a0_5 a1 a1_5
    a2 a2_5 x0 x0_5 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_6 'Design Situation 6' ( ONLY a0 a0_6 a1 a1_6
    a2 a2_6 x0 x0_6 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_7 'Design Situation 7' ( ONLY a0 a0_7 a1 a1_7
    a2 a2_7 x0 x0_7 ntot ntot_3 n10 n10_3 rl rl_3 )
CREATE DESIGN-SITUATION DESSIT_8 'Design Situation 8' ( ONLY a0 a0_8 a1 a1_8
    a2 a2_8 x0 x0_8 ntot ntot_3 n10 n10_3 rl rl_3 )
CREATE DESIGN-SITUATION DESSIT_9 'Design Situation 9' ( ONLY a0 a0_9 a1 a1_9
    a2 a2_9 x0 x0_9 ntot ntot_3 n10 n10_3 rl rl_3 )
CREATE DESIGN-SITUATION DESSIT_10 'Design Situation 10' ( ONLY a0 a0_10 a1
    a1_10 a2 a2_10 x0 x0_1 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_11 'Design Situation 11' ( ONLY a0 a0_11 a1
    a1_11 a2 a2_11 x0 x0_2 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_12 'Design Situation 12' ( ONLY a0 a0_12 a1
    a1_12 a2 a2_12 x0 x0_3 ntot ntot_1 n10 n10_1 rl rl_1 )
CREATE DESIGN-SITUATION DESSIT_13 'Design Situation 13' ( ONLY a0 a0_13 a1
    a1_13 a2 a2_13 x0 x0_4 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_14 'Design Situation 14' ( ONLY a0 a0_14 a1
    a1_14 a2 a2_14 x0 x0_5 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_15 'Design Situation 15' ( ONLY a0 a0_15 a1
    a1_15 a2 a2_15 x0 x0_6 ntot ntot_2 n10 n10_2 rl rl_2 )
CREATE DESIGN-SITUATION DESSIT_16 'Design Situation 16' ( ONLY a0 a0_16 a1
    a1_16 a2 a2_16 x0 x0_7 ntot ntot_3 n10 n10_3 rl rl_3 )
CREATE DESIGN-SITUATION DESSIT_17 'Design Situation 17' ( ONLY a0 a0_17 a1
    a1_17 a2 a2_17 x0 x0_8 ntot ntot_3 n10 n10_3 rl rl_3 )
CREATE DESIGN-SITUATION DESSIT_18 'Design Situation 18' ( ONLY a0 a0_18 a1
    a1_18 a2 a2_18 x0 x0_9 ntot ntot_3 n10 n10_3 rl rl_3 )
%
% Environmental conditions
%
CREATE VARIABLE kc_1 'Char. intercept. Env. cond. 1.' FIXED 6.855e-14
CREATE VARIABLE kc_2 'Char. intercept. Env. cond. 2.' FIXED 1.0666e-13
CREATE VARIABLE sigmae_1 'Stdev of residuals. Env. cond. 1.' FIXED 0.396
CREATE VARIABLE sigmae_2 'Stdev of residuals. Env. cond. 2.' FIXED 0.3
CREATE ENVIRONMENTAL-CONDITION ENVIRO_1 'Environmental condition' ( ONLY
    kc kc_1 sigmae sigmae_1 )
CREATE ENVIRONMENTAL-CONDITION ENVIRO_2 'Environmental condition' ( ONLY
    kc kc_2 sigmae sigmae_2 )
%
% Design cases:
%
CREATE DESIGN-CASE DCA_1 'Des.Case 1' ( ONLY DESSIT_1 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_2 'Des.Case 2' ( ONLY DESSIT_2 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_3 'Des.Case 3' ( ONLY DESSIT_3 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_4 'Des.Case 4' ( ONLY DESSIT_4 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_5 'Des.Case 5' ( ONLY DESSIT_5 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_6 'Des.Case 6' ( ONLY DESSIT_6 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_7 'Des.Case 7' ( ONLY DESSIT_7 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_8 'Des.Case 8' ( ONLY DESSIT_8 ) ( ONLY ENVIRO_1 ) 1.0
CREATE DESIGN-CASE DCA_9 'Des.Case 9' ( ONLY DESSIT_9 ) ( ONLY ENVIRO_1 ) 1.0
```

```
CREATE DESIGN-CASE DCA_10 'Des.Case 10' ( ONLY DESSIT_10 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_11 'Des.Case 11' ( ONLY DESSIT_11 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_12 'Des.Case 12' ( ONLY DESSIT_12 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_13 'Des.Case 13' ( ONLY DESSIT_13 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_14 'Des.Case 14' ( ONLY DESSIT_14 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_15 'Des.Case 15' ( ONLY DESSIT_15 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_16 'Des.Case 16' ( ONLY DESSIT_16 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_17 'Des.Case 17' ( ONLY DESSIT_17 ) ( ONLY ENVIRO_2 ) 1.0
CREATE DESIGN-CASE DCA_18 'Des.Case 18' ( ONLY DESSIT_18 ) ( ONLY ENVIRO_2 ) 1.0
%
% Scope of code:
%
CREATE SCOPE-OF-CODE Scope 'Scope of code' ( ONLY
  DCA_1 DCA_2 DCA_3
  DCA_4 DCA_5 DCA_6
  DCA_7 DCA_8 DCA_9
  DCA_10 DCA_11 DCA_12
  DCA_13 DCA_14 DCA_15
  DCA_16 DCA_17 DCA_18
     )
%
% Code check function - function, variable and code check:
%
CREATE FUNCTION Code_Check 'Code check formula' FORMULA ( ONLY
    ntot      'Design no. of cycles'
    kc        'Char. intercept'
    gm    'Gamma fct. of m+1'
    x0        'Char. bending moment'
    se    'Stdev of residuals'
    emodul    'Mod. of elasticity'
    ws        'Section modulus'
    m         'S-N slope'
    n10       'No. obs. 10-min hist.'
    rl        'Blade length'
    S0  'Safety fac. on load'
    S1   'Safety fac. on load'
    S2   'Safety factor on load'
    S3   'Safety factor on load'
    S4   'Safety factor on strength'
    S5   'Safety factor on strength'
    S6  'Safety factor on strength'     ) (
    '1.0-ntot/kc*gm*(0.5*x0*(0.966*S0+0.048*S1*rl**(0.927*S2)/(n10**(0.211*S3)))*(S4+0.39*S5*s
e**(1.51*S6))/log(ntot)/emodul/ws)**m'
      )
%
CREATE VARIABLE code_check 'Code check function' FUNCTION Code_Check
    ntot kc gammam x0 sigmae emodul ws m n10 rl SafeL0 SafeL1 SafeL2 SafeL3
    SafeS4 SafeS5 SafeS6
CREATE CODE-FUNCTION Code_Check 'Code check function' code_check
CREATE VARIABLE Penalty 'Penalty Function Variable' FUNCTION LindPF1 0 1.5
CREATE PENALTY-FUNCTION Penalty 'Penalty Function' Penalty
CREATE FAILURE-MODE FailMod 'Failure Mode' ( ONLY Code_check ) fail Penalty
    4.265 1.0
CREATE MODE-SET ModeSet 'Set of Failure Modes' ( ONLY FailMod )
ASSIGN DESIGN-PARAMETER STARTING-POINT w DEFAULT 0.0008
ASSIGN DESIGN-PARAMETER OPTIMISATION-BOUNDS w DEFAULT 0.0004 0.002
ASSIGN SAFETY-FACTOR STARTING-POINT SafeL0 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeL1 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeL2 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeL3 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeS4 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeS5 1.0
ASSIGN SAFETY-FACTOR STARTING-POINT SafeS6 1.0
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeL0 0.940 1.100
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeL1 0.900 1.600
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeL2 0.935 1.300
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeL3 0.900 1.200
```

```
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeS4 1.000 1.000
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeS5 0.900 1.200
ASSIGN SAFETY-FACTOR OPTIMISATION-BOUNDS SafeS6 0.900 1.000
DEFINE FORM-SORM BOUNDS OFF
DEFINE FORM-SORM INACTIVE-CONSTRAINTS ON
DEFINE FORM-SORM MULTINORMAL SQP
DEFINE FORM-SORM OPTIMIZATION SQP 40 10 0.0001726335
DEFINE FORM-SORM SENSITIVITY ANALYTICAL ONE-WAY
DEFINE FORM-SORM STARTING-POINT INITIAL ASSIGNED
DEFINE FORM-SORM STARTING-POINT PARAMETER-STUDY PREVIOUS-SOLUTION
DEFINE ANALYSIS-OPTION DIFFERENTIATION 0.001 0.1 0.001 0.001 1.0e-10
DEFINE CODE-CALIBRATION SQP 30 40 0.030 5.0e-04 0.001 1.0e-10
```

Title and authors

Calibration of Partial Safety Factors for Wind Turbine Rotor Blades against Fatigue Failure (in Danish: Kalibrering af partielle sikkerhedsfaktorer for udmattelse af vindmøllerotorer).

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Abstract (max. 2000 characters)

The report describes a calibration of partial safety factors for wind turbine rotor blades subjected to fatigue loading in flapwise and edgewise bending. While earlier models - developed by the authors – dealt with such calibrations for site-specific individual turbines only, the calibration model applied herein covers an integrated analysis with different turbines on different sites and with different blade materials. The result is an optimized set of partial safety factors, i.e. a set of safety factors that lead to minimum deviation from the target reliability of the achieved reliabilities over the selected scope of turbines, sites and materials. The turbines included in the study cover rated powers of 450-600 kW.

Descriptors INIS/EDB

DESIGN; DYNAMIC LOADS; FATIGUE; PROBABILISTIC ESTIMATION; RELIABILITY; ROTORS; SAFETY; WIND TURBINES; CALIBRATION; STANDARDS.